

The Meaning and Measurement of Risk and Return

Learning Objectives

b¹	Define and measure the expected rate of return of an individual investment.	Expected Return Defined and Measured
b²	Define and measure the riskiness of an individual investment.	Risk Defined and Measured
b³	Compare the historical relationship between risk and rates of return in the capital markets.	Rates of Return: The Investor's Experience
b⁴	Explain how diversifying investments affects the riskiness and expected rate of return of a portfolio or combination of assets.	Risk and Diversification
b⁵	Explain the relationship between an investor's required rate of return on an investment and the riskiness of the investment.	The Investor's Required Rate of Return

One of the most important concepts in finance is *risk and return*, which is the total focus of our third principle of finance—**Risk Requires a Reward**. You only have to look at what happened in the stock markets during the past 5 years from 2007 to 2012 to see an overwhelming presence of investment risk. To illustrate, if you had been so unfortunate to buy a portfolio consisting of stocks making up the Standard & Poor's 500 Index in July 2007 and sold in February 2009, you would have lost 54 percent of your investment value. But if you had instead purchased the stock in July 2009 and held until March 2012, you would have a gain of 105 percent. Then if you had held the portfolio of stock for the entire 5 years, 2007–2012, you would still have lost 10 percent of your investment.

Instead of investing in a portfolio of stocks, as represented by the Standard & Poor's 500 Index, you could have chosen to invest in a single stock, such as JP Morgan Chase. If you were so fortunate as to buy the stock in November 2010 and sell 5 months later in April 2011, you would have doubled your money! On the other hand, if you had bought in April 2011, by June 2012 you would have lost half of your investment. Of course, you could have bought Apple at the beginning of 2012 and by June 2012 have a gain of 50 percent. Finally, if you were one of the investors who were eager to buy Facebook stock when it was issued to the public in May 2012, within 3 weeks your investment would have lost 30 percent of its value.

Clearly, owning stocks in recent times has not been for the faint of heart, where in a single day you could have earned as much as 5 percent on your investment, or lost even more.

The market crash and extreme volatility in stock prices in 2008 and 2009 are what Nassim Nicholas Taleb would call a black swan—a highly improbable event that has a massive impact.¹ As a consequence, our confidence in forecasting the future has been undermined. Furthermore, our lives have been impacted and future plans delayed. We are much more cognizant of the financial risks that we face, both individually and in business.

In this chapter, we will help you understand the nature of risk and how risk *should* relate to expected returns on investments. We will look back beyond the past few years to see what we can learn about long-term historical risk-and-return relationships. These are topics that should be of key interest to us all in this day and age.

The need to recognize risk in financial decisions has already been apparent in earlier chapters. In Chapter 2, we referred to the discount rate, or the interest rate, as the opportunity cost of funds, but we did not look at the reasons why that rate might be high or low. For example, we did not explain why in June 2012 you could buy bonds issued by Office Depot that promised to pay a 6.8 percent rate of return, or why you could buy Dole Foods bonds that would give you an 8.0 percent rate of return, provided that both firms make the payments to the investors as promised.

In this chapter, we learn that risk is an integral force underlying rates of return. To begin our study, we define expected return and risk and offer suggestions about how these important concepts of return and risk can be measured quantitatively. We also compare the historical relationship between risk and rates of return. We then explain how diversifying investments can affect the expected return and riskiness of those investments. We also consider how the riskiness of an investment should affect the required rate of return on an investment.

Let's begin our study by looking at what we mean by the expected rate of return and how it can be measured.

¹In his book *The Black Swan: The Impact of the Highly Improbable* (New York: Random House, 2007), Nassim Nicholas Taleb uses the analogy of a black swan, which represents a highly unlikely event. Until black swans were discovered in Australia, everyone assumed that all swans were white. Thus, for Taleb, the black swan symbolizes an event that no one thinks possible.



REMEMBER YOUR PRINCIPLES
Principle This chapter has one primary objective, that of helping you understand **Principle 3: Risk Requires a Reward.**

b¹ Define and measure the expected rate of return of an individual investment.

holding-period return (historical or realized rate of return) the rate of return earned on an investment, which equals the dollar gain divided by the amount invested.

Expected Return Defined and Measured

The expected benefits, or returns, an investment generates come in the form of cash flows. Cash flow, not accounting profit, is the relevant variable the financial manager uses to measure returns. This principle holds true regardless of the type of security, whether it is a debt instrument, preferred stock, common stock, or any mixture of these (such as convertible bonds).

To begin our discussion about an asset's expected rate of return, let's first understand how to compute a **historical** or **realized rate of return** on an investment. It may also be called a **holding-period return**. For instance, consider the dollar return you would have earned had you purchased a share of Google on April 23, 2012, for \$598.45 and sold it less than 1 month later on May 17 for \$637.85. The dollar return on your investment would have been \$39.40 (\$39.40 = \$637.85 – \$598.45), assuming that the company paid no dividend. In addition to the dollar gain, we can calculate the rate of return as a percentage. It is useful to summarize the return on an investment in terms of a percentage because that way we can see the return per dollar invested, which is independent of how much we actually invest.

The rate of return earned from the investment in Google can be calculated as the ratio of the dollar return of \$39.40 divided by your \$598.45 investment in the stock, or 6.58 percent (6.58% = 0.0658 = \$39.40 ÷ \$598.45).

We can formalize the return calculations using equations (6-1) and (6-2):

Holding-period dollar gain would be:

$$\text{Holding-period dollar gain, DG} = \text{price}_{\text{end of period}} + \frac{\text{cash distribution (dividend)}}{\text{price}_{\text{beginning of period}}} - \text{price}_{\text{beginning of period}} \quad (6-1)$$

and for the holding-period rate of return

$$\text{Holding-period rate of return, } r = \frac{\text{dollar gain}}{\text{price}_{\text{beginning of period}}} = \frac{\text{price}_{\text{end of period}} + \text{dividend} - \text{price}_{\text{beginning of period}}}{\text{price}_{\text{beginning of period}}} \quad (6-2)$$

The method we have just used to compute the holding-period return on our investment in Google tells us the return we actually earned during a historical time period. However, the risk–return trade-off that investors face on a day-to-day basis is based *not* on realized rates of return but on what the investor *expects* to earn on an investment in the future. We can think of the rate of return that will ultimately be realized from making a risky investment in terms of a range of possible return outcomes, much like the distribution of grades for this class at the end of the term. To describe this range of possible returns, it is customary to use the average of the different possible returns. We refer to the average of the possible rates of return as the investment's **expected rate of return**.

Accurately measuring expected future cash flows is not easy in a world of uncertainty. To illustrate, assume you are considering an investment costing \$10,000, for which the future cash flows from owning the security depend on the state of the economy, as estimated in Table 6-1.

expected rate of return The arithmetic mean or average of all possible outcomes where those outcomes are weighted by the probability that each will occur.

TABLE 6-1 Measuring the Expected Return of an Investment

State of the Economy	Probability of the States ^a	Cash Flows from the Investment	Percentage Returns (Cash Flow ÷ Investment Cost)
Economic recession	20%	\$1,000	10% (\$1,000 ÷ \$10,000)
Moderate economic growth	30%	1,200	12% (\$1,200 ÷ \$10,000)
Strong economic growth	50%	1,400	14% (\$1,400 ÷ \$10,000)

^aThe probabilities assigned to the three possible economic conditions have to be determined subjectively, which requires managers to have a thorough understanding of both the investment cash flows and the general economy.

In any given year, the investment could produce any one of three possible cash flows, depending on the particular state of the economy. With this information, how should we select the cash flow estimate that means the most for measuring the investment's expected rate of return? One approach is to calculate an *expected* cash flow. The expected cash flow is simply the weighted average of the *possible* cash flow outcomes such that the weights are the probabilities of the various states of the economy occurring. Stated as an equation:

$$\begin{aligned} \text{Expected cash flow, } \overline{CF} = & \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 1} & \text{of state 1} \\ (CF_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 2} & \text{of state 2} \\ (CF_2) & (Pb_2) \end{array} \right) \\ & + \dots + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 3} & \text{of state 3} \\ (CF_3) & (Pb_3) \end{array} \right) \end{aligned} \quad (6-3)$$

For the present illustration:

$$\begin{aligned} \text{Expected cash flow} &= (0.2)(\$1,000) + (0.3)(\$1,200) \\ &\quad + (0.5)(\$1,400) \\ &= \$1,260 \end{aligned}$$

In addition to computing an expected dollar return from an investment, we can also calculate an expected rate of return earned on the \$10,000 investment. Similar to the expected cash flow, the expected rate of return is *a weighted average of all the possible returns, weighted by the probability that each return will occur*. As the last column in Table 6-1 shows, the \$1,400 cash inflow, assuming strong economic growth, represents a 14 percent return (\$1,400 ÷ \$10,000). Similarly, the \$1,200 and \$1,000 cash flows result in 12 percent and 10 percent returns, respectively. Using these percentage returns in place of the dollar amounts, the expected rate of return can be expressed as follows:

$$\begin{aligned} \text{Expected rate of return, } \bar{r} = & \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 1} & \text{of state 1} \\ (r_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 2} & \text{of state 2} \\ (r_2) & (Pb_2) \end{array} \right) \\ & + \dots + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 3} & \text{of state 3} \\ (r_3) & (Pb_3) \end{array} \right) \end{aligned} \quad (6-4)$$

In our example:

$$\bar{r} = (0.2)(10\%) + (0.3)(12\%) + (0.5)(14\%) = 12.6\%$$

REMEMBER YOUR PRINCIPLES
Principle Remember that future cash flows, not reported earnings, determine the investor's rate of return. That is, **Principle 1: Cash Flow Is What Matters.**

CAN YOU DO IT?

COMPUTING EXPECTED CASH FLOW AND EXPECTED RETURN

You are contemplating making a \$5,000 investment that would have the following possible outcomes in cash flow each year. What is the expected value of the future cash flows and the expected rate of return?

PROBABILITY	CASH FLOW
0.30	\$350
0.50	625
0.20	900

(The solution can be found on page 187.)

To this point, we have learned how to calculate a *historical* holding-period return, expressed both in dollars and percentages. Also, we have seen how to estimate dollar and percentage returns that we expect to earn in the future. These financial tools may be summarized as follows:

FINANCIAL DECISION TOOLS		
Name of Tool	Formula	What It Tells You
Holding-period dollar gain	$\text{Holding-period dollar gain, DG} = \text{price}_{\text{end of period}} + \text{cash distribution (dividend)} - \text{price}_{\text{beginning of period}}$	Measures the dollar gain on an investment for a period of time
Holding-period rate of return	$\text{Holding-period rate of return, } r = \frac{\text{dollar gain}}{\text{price}_{\text{beginning of period}}} = \frac{\text{price}_{\text{end of period}} + \text{dividend} - \text{price}_{\text{beginning of period}}}{\text{price}_{\text{beginning of period}}}$	Calculates the percentage rate of return for a security held for a period of time
Expected cash flow	$\text{Expected cash flow, } \overline{CF} = \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 1} & \text{of state 1} \\ (CF_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 2} & \text{of state 2} \\ (CF_2) & (Pb_2) \end{array} \right) + \dots + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 3} & \text{of state 3} \\ (CF_3) & (Pb_3) \end{array} \right)$	Estimates the cash flows that can be expected from an investment, recognizing that there are multiple possible outcomes from the investment
Expected rate of return	$\text{Expected rate of return, } \bar{r} = \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 1} & \text{of state 1} \\ (r_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 2} & \text{of state 2} \\ (r_2) & (Pb_2) \end{array} \right) + \dots + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 3} & \text{of state 3} \\ (r_3) & (Pb_3) \end{array} \right)$	An estimate of the expected rate of return on an investment, recognizing that there are multiple possible outcomes from the investment

With our concept and measurement of expected returns, let's consider the other side of the investment coin: risk.

Concept Check

1. When we speak of “benefits” from investing in an asset, what do we mean?
2. Why is it difficult to measure future cash flows?
3. Define *expected rate of return*.

b² Define and measure the riskiness of an individual investment.

Risk Defined and Measured

Because we live in a world where events are uncertain, the way we see risk is vitally important in almost all dimensions of our life. The Greek poet and statesman Solon, writing in the sixth century B.C., put it this way:

There is risk in everything that one does, and no one knows where he will make his landfall when his enterprise is at its beginning. One man, trying to act effectively, fails to foresee something and falls into great and grim ruination, but to another man, one who is acting ineffectively, a god gives good fortune in everything and escape from his folly.²

²From *Iambi et Elegi Graeci ante Alexandrum Cantati*, vol. 2. Edited by M.L. West, translated by Arthur W.H. Adkins. (Oxford: Clarendon Press, 1972).

DID YOU GET IT?

COMPUTING EXPECTED CASH FLOW AND EXPECTED RETURN

PROBABILITY	POSSIBLE		EXPECTED	
	CASH FLOW	RETURN	CASH FLOW	RETURN
0.30	\$350	7.0%	\$105.00	2.1%
0.50	625	12.5%	312.50	6.3%
0.20	900	18.0%	180.00	3.6%
Expected cash flow and rate of return			\$597.50	12.0%

The possible returns are equal to the possible cash flows divided by the \$5,000 investment. The expected cash flow and return are equal to the possible cash flows and possible returns multiplied by the probabilities.

Solon would have given more of the credit to Zeus than we would for the outcomes of our ventures. However, his insight reminds us that little is new in this world, including the need to acknowledge and compensate as best we can for the risks we encounter. In fact, the significance of risk and the need for understanding what it means in our lives is noted by Peter Bernstein in the following excerpt:

What is it that distinguishes the thousands of years of history from what we think of as modern times? The answer goes way beyond the progress of science, technology, capitalism, and democracy.

The distant past was studded with brilliant scientists, mathematicians, inventors, technologists, and political philosophers. . . . [T]he skies had been mapped, the great library of Alexandria built, and Euclid's geometry taught. . . . Coal, oil, iron, and copper have been at the service of human beings for millennia. . . .

The revolutionary idea that defines the boundary between modern times and the past is the mastery of risk. . . . Until human beings discovered a way across that boundary, the future was a mirror of the past or the murky domain of oracles and soothsayers. . . .³

In our study of risk, we want to consider these questions:

1. What is risk?
2. How do we know the amount of risk associated with a given investment; that is, how do we measure risk?
3. If we choose to diversify our investments by owning more than one asset, as most of us do, will such diversification reduce the riskiness of our combined portfolio of investments?

Without intending to be trite, risk means different things to different people, depending on the context and on how they feel about taking chances. For the student, risk is the possibility of failing an exam or the chance of not making his or her best grades. For the coal miner or the oil field worker, risk is the chance of an explosion in the mine or at the well site. For the retired person, risk means perhaps not being able to live comfortably on a fixed income. For the entrepreneur, risk is the chance that a new venture will fail.

While certainly acknowledging these different kinds of risk, we limit our attention to the risk inherent in an investment. In this context, **risk** is the *potential variability in future cash flows*. The wider the range of possible events that can occur, the greater the risk.⁴ If we think about it, this is a relatively intuitive concept.

risk potential variability in future cash flows.

³From "Introduction" in *Against the Gods: The Remarkable Story of Risk* by Peter Bernstein, published by John Wiley & Sons, Inc., New York: 1996.

⁴When we speak of possible events, we must not forget that it is the highly unlikely event that we cannot anticipate that may have the greatest impact on the outcome of an investment. So, we evaluate investment opportunities based on the best information available, but there may be a *black swan* that we cannot anticipate.

To help us grasp the fundamental meaning of risk within this context, consider two possible investments:

1. The first investment is a U.S. Treasury bond, a government security that matures in 10 years and promises to pay an annual return of 2 percent. If we purchase and hold this security for 10 years, we are virtually assured of receiving no more and no less than 2 percent on an annualized basis. For all practical purposes, the risk of loss is nonexistent.
2. The second investment involves the purchase of the stock of a local publishing company. Looking at the past returns of the firm's stock, we have made the following estimate of the annual returns from the investment:

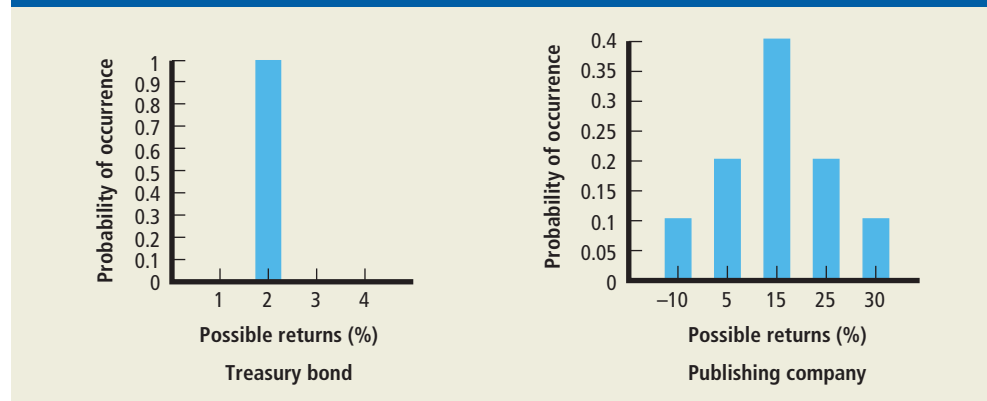
CHANCE OF OCCURRENCE	RATE OF RETURN ON INVESTMENT
1 chance in 10 (10% chance)	−10%
2 chances in 10 (20% chance)	5%
4 chances in 10 (40% chance)	15%
2 chances in 10 (20% chance)	25%
1 chance in 10 (10% chance)	30%

Investing in the publishing company could conceivably provide a return as high as 30 percent if all goes well, or a loss of 10 percent if everything goes against the firm. However, in future years, both good and bad, we could expect a 14 percent return on average.⁵

$$\begin{aligned}
 \text{Expected return} &= (0.10)(-10\%) + (0.20)(5\%) + (0.40)(15\%) + (0.20)(25\%) \\
 &\quad + (0.10)(30\%) \\
 &= 14\%
 \end{aligned}$$

Comparing the Treasury bond investment with the publishing company investment, we see that the Treasury bond offers an expected 2 percent annualized rate of return, whereas the publishing company has a much higher expected rate of return of 14 percent. However, our investment in the publishing firm is clearly more “risky”—that is, there is greater uncertainty about the final outcome. Stated somewhat differently, there is a greater variation or dispersion of possible returns, which in turn implies greater risk.⁶ Figure 6-1 shows these differences graphically in the form of discrete probability distributions.

FIGURE 6-1 The Probability Distribution of the Returns on Two Investments



⁵We assume that the particular outcome or return earned in 1 year does *not* affect the return earned in the subsequent year. Technically speaking, the distribution of returns in any year is assumed to be independent of the outcome in any prior year.

⁶How can we possibly view variations above the expected return as risk? Should we even be concerned with the positive deviations above the expected return? Some would say “no,” viewing risk as only the negative variability in returns from a predetermined minimum acceptable rate of return. However, as long as the distribution of returns is symmetrical, the same conclusions will be reached.

Although the return from investing in the publishing firm is clearly less certain than for Treasury bonds, quantitative measures of risk are useful when the difference between two investments is not so evident. We can quantify the risk of an investment by computing the variance in the possible investment returns and its square root, the **standard deviation** (σ). For the case where there are n possible returns (that is, states of the economy), we calculate the variance as follows:

$$\begin{aligned} \text{Variance in rates of return } (\sigma^2) = & \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 1} & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 1} \end{array} \right] \\ & + \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 2} & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 2} \end{array} \right] \\ & + \cdots + \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state } n & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state } n \end{array} \right] \end{aligned} \quad (6-5)$$

standard deviation a statistical measure of the spread of a probability distribution calculated by squaring the difference between each outcome and its expected value, weighting each value by its probability, summing over all possible outcomes, and taking the square root of this sum.

For the publishing company's common stock, we calculate the standard deviation using the following five-step procedure:

- STEP 1** Calculate the expected rate of return of the investment, which was calculated above to be 14 percent.
- STEP 2** Subtract the expected rate of return of 14 percent from each of the possible rates of return and square the difference.
- STEP 3** Multiply the squared differences calculated in step 2 by the probability that those outcomes will occur.
- STEP 4** Sum all the values calculated in step 3 together. The sum is the variance of the distribution of possible rates of return. Note that the variance is actually the *average squared difference between the possible rates of return and the expected rate of return*.
- STEP 5** Take the square root of the variance calculated in step 4 to calculate the standard deviation of the distribution of possible rates of return.

Table 6-2 illustrates the application of this process, which results in an estimated standard deviation for the common stock investment of 11.14 percent. This compares to the Treasury bond investment, which is risk-free, and has a standard deviation of zero percent. The more risky the investment, the higher is its standard deviation.

TABLE 6-2 Measuring the Variance and Standard Deviation of the Publishing Company Investment

State of the World	Rate of Return	Chance or Probability		Step 2	Step 3
A	B	C	D = B × C	E = (B - $\bar{r}$) ²	F = E × C
1	-10%	0.10	-1%	576%	57.6%
3	5%	0.20	1%	81%	16.2%
4	15%	0.40	6%	1%	0.40%
4	25%	0.20	5%	121%	24.2%
5	30%	0.10	3%	256%	25.6%

Step 1: Expected Return ($\bar{r}$) = —————→ 14%

Step 4: Variance = —————→ 124%

Step 5: Standard Deviation = —————→ 11.14%

CAN YOU DO IT?

COMPUTING THE STANDARD DEVIATION

In the preceding “Can You Do It?” on page 185, we computed the expected cash flow of \$597.50 and the expected return of 12 percent on a \$5,000 investment. Now let’s calculate the standard deviation of the returns. The probabilities of possible returns are given as follows:

PROBABILITY	RETURNS
0.30	7.0%
0.50	12.5%
0.20	18.0%

(The solution can be found on page 193.)

Alternatively, we could use equation (6-5) to calculate the standard deviation in investment returns as follows:

$$\begin{aligned}\sigma &= \left[\begin{aligned} &(-10\% - 14\%)^2(0.10) + (5\% - 14\%)^2(0.20) \\ &+ (15\% - 14\%)^2(0.40) + (25\% - 14\%)^2(0.20) \\ &+ (30\% - 14\%)^2(0.10) \end{aligned} \right]^{\frac{1}{2}} \\ &= \sqrt{124\%} = 11.14\% \end{aligned}$$

Although the standard deviation of returns provides us with a quantitative measure of an asset’s riskiness, how should we interpret the result? What does it mean? Is the 11.14 percent standard deviation for the publishing company investment good or bad? First, we should remember that statisticians tell us that two-thirds of the time, an event will fall within 1 standard deviation of the expected value (assuming the distribution is normally distributed; that is, it is shaped like a bell). Thus, given a 14 percent expected return and a standard deviation of 11.14 percent for the publishing company investment, we can reasonably anticipate that the actual returns will fall between 2.86 percent and 25.14 percent ($14\% \pm 11.14\%$) two-thirds of the time. In other words, there is not much certainty with this investment.

A second way of answering the question about the meaning of the standard deviation comes by comparing the investment in the publishing firm against other investments. Obviously the attractiveness of a security with respect to its return and risk cannot be determined in isolation. Only by examining other available alternatives can we reach a conclusion about a particular investment’s risk. For example, if another investment, say, an investment in a firm that owns a local radio station, has the same expected return as the publishing company, 14 percent, but with a standard deviation of 7 percent, we would consider the risk associated with the publishing firm, 11.14 percent, to be excessive. In the technical jargon of modern portfolio theory, the radio station investment is said to “dominate” the publishing firm investment. In commonsense terms, this means that the radio station investment has the same expected return as the publishing company investment but is less risky.

FINANCE AT WORK

A DIFFERENT PERSPECTIVE OF RISK

The first Chinese symbol shown here represents danger; the second stands for opportunity. The Chinese define risk as the combination of danger and opportunity. Greater risk, according to the Chinese, means we have greater opportunity to do well but also greater danger of doing badly.

危機

What if we compare the investment in the publishing company with one in a quick oil-change franchise, an investment in which the expected rate of return is an attractive 24 percent but the standard deviation is estimated at 13 percent? Now what should we do? Clearly, the oil-change franchise has a higher expected rate of return, but it also has a larger standard deviation. In this example, we see that the real challenge in selecting the better investment comes when one investment has a higher expected rate of return but also exhibits greater risk. *Here the final choice is determined by our attitude toward risk, and there is no single right answer.* You might select the publishing company, whereas I might choose the oil-change investment, and neither of us would be wrong. We would simply be expressing our tastes and preferences about risk and return.

To summarize, the riskiness of an investment is of primary concern to an investor, where the standard deviation is the conventional measure of an investment's riskiness. This decision tool is represented as follows:

FINANCIAL DECISION TOOLS		
Name of Tool	Formula	What It Tells You
Standard deviation in rates of return $= \sqrt{\text{variance}}$	$\begin{aligned} \text{Variance in rates of return } (\sigma^2) &= \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 1} & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 1} \end{array} \right] \\ &+ \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 2} & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 2} \end{array} \right] \\ &+ \dots + \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state } n & \text{of return} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state } n \end{array} \right] \end{aligned}$	A measure of risk, as determined by the square root of the variance of cash flows or rates of returns, which measures the volatility of cash flows or returns

EXAMPLE 6.1**Computing the expected return and standard deviation**

You are considering two investments, X and Y. The distributions of possible returns are shown below:

PROBABILITY	POSSIBLE RETURNS	
	INVESTMENT X	INVESTMENT Y
0.05	−10%	0%
0.25	5%	5%
0.40	20%	16%
0.25	30%	24%
0.05	40%	32%

Compute the expected return and standard deviation for each investment. Do you have a preference for one investment over the other if you were making the decision?

STEP 1: FORMULATE A SOLUTION STRATEGY

To compute the expected return for each investment, equation (6-4) is used, where there are five possible outcomes or states:

$$\begin{aligned} \text{Expected rate of return, } \bar{r} &= \left(\begin{array}{cc} \text{rate of return} & \text{probability of} \\ \text{for state 1} & \text{state 1} \end{array} \right) \times \begin{array}{c} (r_1) \\ (Pb_1) \end{array} \\ &+ \left(\begin{array}{cc} \text{rate of return} & \text{probability of} \\ \text{for state 2} & \text{state 2} \end{array} \right) \times \begin{array}{c} (r_2) \\ (Pb_2) \end{array} + \dots + \left(\begin{array}{cc} \text{rate of return} & \text{probability of} \\ \text{for state 5} & \text{state 5} \end{array} \right) \times \begin{array}{c} (r_5) \\ (Pb_5) \end{array} \end{aligned}$$

$$\begin{aligned} \text{Variance in} \\ \text{rates of return} \\ (\sigma^2) &= \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 1} & \text{of return} \\ (r_1) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 1} \\ (Pb_1) \end{array} \right] \\ &+ \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 2} & \text{of return} \\ (r_2) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 2} \\ (Pb_2) \end{array} \right] \\ &+ \cdots + \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state } n & \text{of return} \\ (r_n) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state } n \\ (Pb_n) \end{array} \right] \end{aligned}$$

Standard deviation (σ) = $\sqrt{\text{variance}}$

STEP 2: CRUNCH THE NUMBERS

For investment X:

$$\begin{aligned}\text{Expected rate of return, } \bar{r} &= (0.05)(-10\%) + (0.25)(5\%) + (0.40)(20\%) \\ &\quad + (0.25)(30\%) + (0.05)(40\%) \\ &= 18.25\%\end{aligned}$$

$$\begin{aligned} \text{Variance in} \\ \text{rates of returns} &= (0.05)(-10\% - 18.25\%)^2 + (0.25)(5\% - 18.25\%)^2 \\ (\sigma^2) &\quad + (0.40)(20\% - 18.25\%)^2 + (0.25)(30\% - 18.25\%)^2 \\ &\quad + (0.05)(40\% - 18.25\%)^2 \\ &= 143.19\% \end{aligned}$$

Standard deviation of
rates of returns
(σ)

$$= \sqrt{\text{variance}} = \sqrt{143.19\%} = 11.97\%.$$

For investment Y:

$$\begin{aligned}\text{Expected rate of return, } \bar{r} &= (0.05)(0\%) + (0.25)(5\%) + (0.40)(16\%) + (0.25)(24\%) \\ &\quad + (0.05)(32\%) \\ &= 15.25\%\end{aligned}$$

$$\begin{aligned} \text{Variance in} \\ \text{rates of returns } (\sigma^2) &= (0.05)(0\% - 15.25\%)^2 + (0.25)(5\% - 15.25\%)^2 \\ &\quad + (0.40)(16\% - 15.25\%)^2 + (0.25)(24\% - 15.25\%)^2 \\ &\quad + (0.05)(32\% - 15.25\%)^2 \\ &= 71.29\% \end{aligned}$$

Standard deviation of
rates of returns
(σ)

$$= \sqrt{\text{variance}} = \sqrt{71.29\%} = 8.44\%$$

STEP 3: ANALYZE YOUR RESULTS

The results are as follows:

INVESTMENT	EXPECTED RETURN	STANDARD DEVIATION
Investment X	18.25%	11.97%
Investment Y	15.25%	8.44%

In this case, you will have to take on more risk if you want additional expected return. (There is no money tree.) Thus, the choice depends on the investor's preference for risk and return. There is no single right answer.

DID YOU GET IT?

COMPUTING THE STANDARD DEVIATION

DEVIATION (POSSIBLE RETURN – 12% EXPECTED RETURN)	DEVIATION SQUARED	PROBABILITY	PROBABILITY × DEVIATION SQUARED
–5.0%	25.00%	0.30	7.500%
0.5%	0.25%	0.50	0.125%
6.0%	36.00%	0.20	7.200%
Sum of squared deviations × probability (variance)			14.825%
Standard deviation			3.85%

Concept Check

1. How is risk defined?
2. How does the standard deviation help us measure the riskiness of an investment?
3. Does greater risk imply a bad investment?

Rates of Return: The Investor's Experience

So far, we have mostly used hypothetical examples of expected rates of return and risk; however, it is also interesting to look at returns that investors have actually received on different types of securities. For example, Ibbotson Associates publishes the long-term historical annual rates of return for the following types of investments beginning in 1926 and continuing to the present:

1. Common stocks of large companies
2. Common stocks of small firms
3. Long-term corporate bonds
4. Long-term U.S. government bonds
5. Intermediate-term U.S. government bonds
6. U.S. Treasury bills (short-term government securities)

Before comparing these returns, we should think about what to expect. First, we would intuitively expect a Treasury bill (short-term government securities) to be the least risky of the six portfolios. Because a Treasury bill has a short-term maturity date, the price is less volatile (less risky) than the price of an intermediate- or long-term government security. In turn, because there is a chance of default on a corporate bond, which is essentially nonexistent for government securities, a long-term government bond is less risky than a long-term corporate bond. Finally, the common stocks of large companies are more risky than corporate bonds, with small-company stocks being more risky than large-firm stocks.

With this in mind, we could reasonably expect different rates of return to the holders of these varied securities. If the market rewards an investor for assuming risk, the average annual rates of return should increase as risk increases.

A comparison of the annual rates of return for five portfolios and the inflation rate for the years 1926 to 2011 is provided in Figure 6-2. Four aspects of these returns are included: (1) the nominal average annual rate of return; (2) the standard deviation of the returns, which measures the volatility, or riskiness, of the returns; (3) the real average annual rate of return, which is the nominal return less the inflation rate; and (4) the risk premium, which represents the additional return received beyond the risk-free rate (Treasury bill rate) for assuming risk. Looking at the



3 Compare the historical relationship between risk and rates of return in the capital markets.

FIGURE 6-2 Historical Rates of Return

Securities	Nominal Average Annual Returns	Standard Deviation of Returns	Real Average Annual Returns ^a	Risk Premium ^b
Large-company stocks	11.8%	20.3%	8.7%	8.0%
Small-company stocks	16.5%	32.5%	13.4%	12.7%
Corporate bonds	6.4%	8.4%	3.3%	2.6%
Intermediate-term government bonds	5.5%	5.7%	3.0%	2.3%
U.S. Treasury bills	3.8%	3.1%	0.7%	0.0%
Inflation	3.1%	4.2%		

^aThe real return equals the nominal returns less the inflation rate of 3.1 percent.

^bThe risk premium equals the nominal security return less the average risk-free rate (Treasury bills) of 3.8 percent.

Source: Data from Summary Statistics of Annual Total Returns: 1926 to 2011 Yearbook. Copyright © 2012 Ibbotson Associates Inc.

first two columns of nominal average annual returns and standard deviations, we get a good overview of the risk–return relationships that have existed over the 86 years ending in 2011. In every case, there has been a positive relationship between risk and return, with Treasury bills being least risky and small-company stocks being most risky.

The return information in Figure 6-2 clearly demonstrates that only common stock has in the long run served as an inflation hedge and provided any substantial risk premium. However, it is equally apparent that the common stockholder is exposed to a sizable amount of risk, as is demonstrated by the 20.3 percent standard deviation for large-company stocks and the 32.5 percent standard deviation for small-company stocks. In fact, in the 1926 to 2011 time frame, common shareholders of large firms received negative returns in 22 of the 86 years, compared with only 1 (1938) in 86 years for Treasury bills.

Concept Check

1. What is the additional compensation for assuming greater risk called?
2. In Figure 6-2, which rate of return is the risk-free rate? Why?

b4 Explain how diversifying investments affects the riskiness and expected rate of return of a portfolio or combination of assets.

Risk and Diversification

More can be said about risk, especially about its nature, when we own more than one asset in our investment portfolio. Let's consider for the moment how risk is affected if we diversify our investment by holding a variety of securities.

Let's assume that you graduated from college in December 2011. Not only did you get the good job you had hoped for but you also finished the year with a little nest egg—not enough to take that summer fling to Europe like some of your college friends but a nice surplus. Besides, you suspected that they used credit cards to go anyway. So right after graduation, you used some of your nest egg to buy Harley-Davidson stock. Then a couple of months later, you purchased Starbucks stock. (As the owner of a Harley Softail motorcycle, you've had a passion for riding since your high school days. And you give Starbucks credit for helping you get through many long study sessions.) But since making these investments, you have focused on your career and seldom thought about your investments. Your first extended break from work came in June 2012. After a lazy Saturday morning, you decided to get on the Internet to see how your investments had done over the previous several months. You bought Harley-Davidson for \$37, and the stock was now trading at almost \$50. “Not

bad,” you thought. But then the bad news—Starbucks was selling for \$54, compared to the \$62 that you paid for the stock.

Clearly, what we have described about Harley-Davidson and Starbucks were events unique to these two companies, and as we would expect, investors reacted accordingly; that is, the value of the stock changed in light of the new information. Although we might have wished we had owned only Starbucks stock at the time, most of us would prefer to avoid such uncertainties; that is, we are risk averse. Instead, we would like to reduce the risk associated with our investment portfolio, without having to accept a lower expected return. Good news: It is possible by diversifying your portfolio!

Diversifying Away the Risk

If we diversify our investments across different securities rather than invest in only one stock, the variability in the returns of our portfolio should decline. The reduction in risk will occur if the stock returns within our portfolio do not move precisely together over time—that is, if they are not perfectly correlated. Figure 6-3 shows graphically what we could expect to happen to the variability of returns as we add additional stocks to the portfolio. The reduction occurs because some of the volatility in returns of a stock are unique to that security. The unique variability of a single stock tends to be countered by the uniqueness of another security. However, we should not expect to eliminate all risk from our portfolio. In practice, it would be rather difficult to cancel all the variations in returns of a portfolio, because stock prices have some tendency to move together. Thus, we can divide the total risk (total variability) of our portfolio into two types of risk: (1) **company-unique risk**, or **unsystematic risk**, and (2) **market risk**, or **systematic risk**. Company-unique risk might also be called **diversifiable risk**, in that it *can be diversified away*. Market risk is **nondiversifiable risk**; it *cannot be eliminated through random diversification*. These two types of risk are shown graphically in Figure 6-3. Total risk declines until we have approximately 20 securities, and then the decline becomes very slight.

The remaining risk, which would typically be about 40 percent of the total risk, is the portfolio’s systematic, or market, risk. At this point, our portfolio is highly correlated with all securities in the marketplace. Events that affect our portfolio now are not so much unique events as changes in the general economy, major political events, and sociological changes. Examples include changes in interest rates in the economy, changes in tax legislation that affect all companies, or increasing public concern about the effect of business practices on the environment. Our measure of risk should, therefore, measure how responsive a stock or portfolio is to changes in a market portfolio, such as the New York Stock Exchange or the S&P 500 Index.⁷

company-unique risk see unsystematic risk.

unsystematic risk the risk related to an investment return that can be eliminated through diversification. Unsystematic risk is the result of factors that are unique to the particular firm. Also called company-unique risk or diversifiable risk.

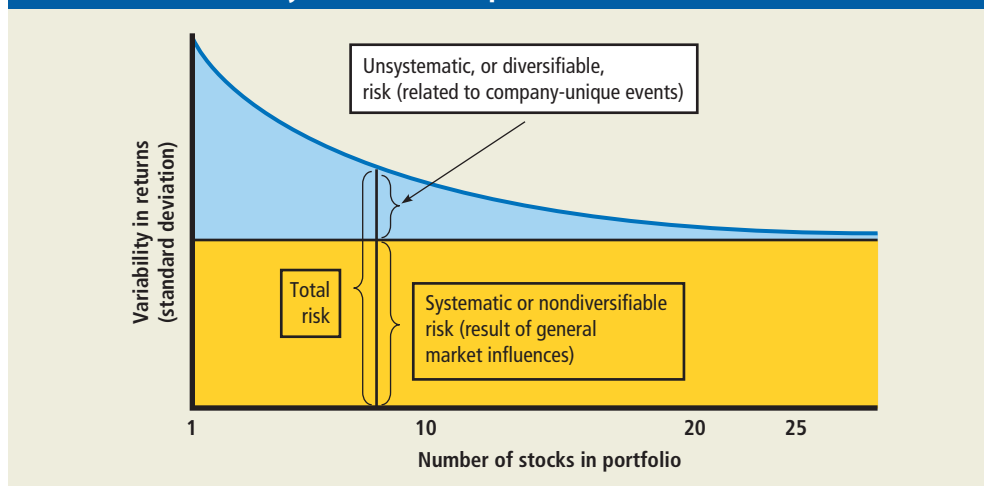
market risk see systematic risk.

systematic risk (1) the risk related to an investment return that cannot be eliminated through diversification. Systematic risk results from factors that affect all stocks. Also called market risk or nondiversifiable risk. (2) The risk of a project from the viewpoint of a well-diversified shareholder. This measure takes into account that some of the project’s risk will be diversified away as the project is combined with the firm’s other projects, and, in addition, some of the remaining risk will be diversified away by shareholders as they combine this stock with other stocks in their portfolios.

diversifiable risk see unsystematic risk.

nondiversifiable risk see systematic risk.

FIGURE 6-3 Variability of Returns Compared with Size of Portfolio



⁷The New York Stock Exchange Index is an index that reflects the performance of all stocks listed on the New York Stock Exchange. The Standard & Poor’s (S&P) 500 Index is similarly an index that measures the combined stock-price performance of the companies that constitute the 500 largest companies in the United States, as designated by Standard & Poor’s.

Measuring Market Risk

To help clarify the idea of systematic risk, let's examine the relationship between the common stock returns of Home Depot and the returns of the S&P 500 Index.

The monthly returns for Home Depot and the S&P 500 Index for the 12 months ending June 2012 are presented in Table 6-3 and Figure 6-4. These *monthly returns*, or *holding-period returns*, as they are often called, are calculated as follows.⁸

$$\begin{aligned}\text{Monthly holding return} &= \frac{\text{price}_{\text{end of month}} - \text{price}_{\text{beginning of month}}}{\text{price}_{\text{beginning of month}}} \\ &= \frac{\text{price}_{\text{end of month}}}{\text{price}_{\text{beginning of month}}} - 1\end{aligned}\quad (6-6)$$

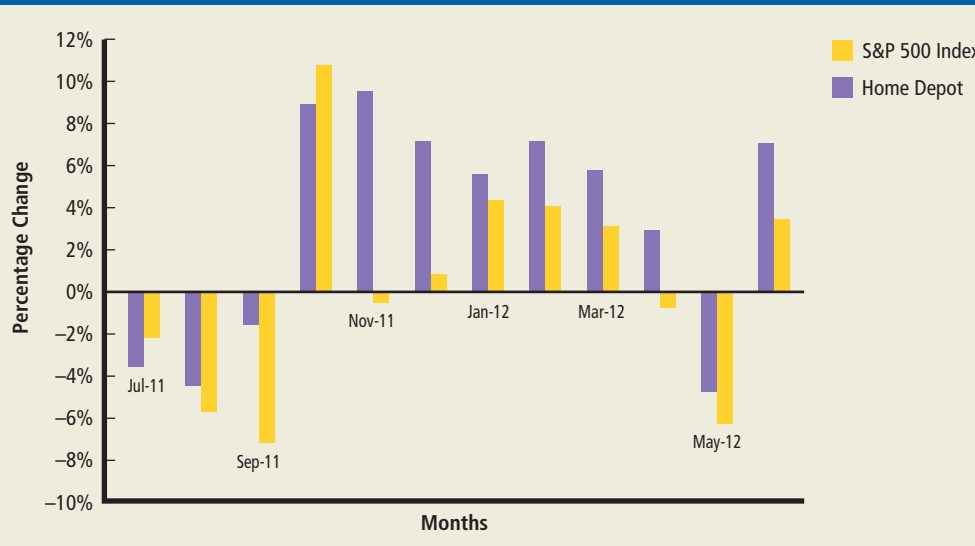
For instance, the holding-period return for Home Depot and the S&P 500 Index for January 2012 is computed as follows:

$$\begin{aligned}\text{The Home Depot return} &= \frac{\text{stock price at end of January 2012}}{\text{stock price at end of December 2011}} - 1 \\ &= \frac{\$44.39}{\$42.04} - 1 = 0.0559 = 5.59\%\end{aligned}$$

$$\begin{aligned}\text{The S\&P 500 Index return} &= \frac{\text{index value at end of January 2012}}{\text{index value at end of December 2011}} - 1 \\ &= \frac{\$1,312.41}{\$1,257.60} - 1 = 0.0436 = 4.36\%\end{aligned}$$

At the bottom of Table 6-3, we have also computed the averages of the returns for the 12 months, for both Home Depot and the S&P 500 Index, and the standard deviation for these returns. Because we are using historical return data, we assume each observation has

FIGURE 6-4 Monthly Holding-Period Returns: Home Depot versus the S&P 500 Index, July 2011 Through June 2012



Source: Data from Yahoo Finance

⁸For simplicity's sake, we are ignoring the dividend that the investor receives from the stock as part of the total return. In other words, letting D_t equal the dividend received by the investor in month t , the holding-period return would more accurately be measured as

$$r_1 = \frac{P_t + D_t}{P_{t-1}} - 1 \quad (6-7)$$

TABLE 6-3 Monthly Holding-Period Returns, Home Depot versus the S&P 500 Index, June 2011 Through June 2012

Month and Year	Home Depot		S&P 500 Index	
	Price	Returns	Price	Returns
2011				
June	\$36.22		\$1,320.64	
July	34.93	−3.56%	1,292.28	−2.15%
August	33.38	−4.44%	1,218.89	−5.68%
September	32.87	−1.53%	1,131.42	−7.18%
October	35.80	8.91%	1,253.30	10.77%
November	39.22	9.55%	1,246.96	−0.51%
December	42.04	7.19%	1,257.60	0.85%
2012				
January	44.39	5.59%	1,312.41	4.36%
February	47.57	7.16%	1,365.68	4.06%
March	50.31	5.76%	1,408.47	3.13%
April	51.79	2.94%	1,397.91	−0.75%
May	49.34	−4.73%	1,310.33	−6.27%
June	52.83	7.07%	1,355.69	3.46%
Average return		3.33%		0.34%
Standard deviation		5.40%		5.23%

Source: Data from Yahoo Finance

an equal probability of occurrence. Thus, the average holding-period return is found by summing the returns and dividing by the number of months; that is,

$$\text{Average holding-period return} = \frac{\text{return in month 1} + \text{return in month 2} + \cdots + \text{return in last month}}{\text{number of monthly returns}} \quad (6-8)$$

and the standard deviation is computed as follows:

Standard
deviation

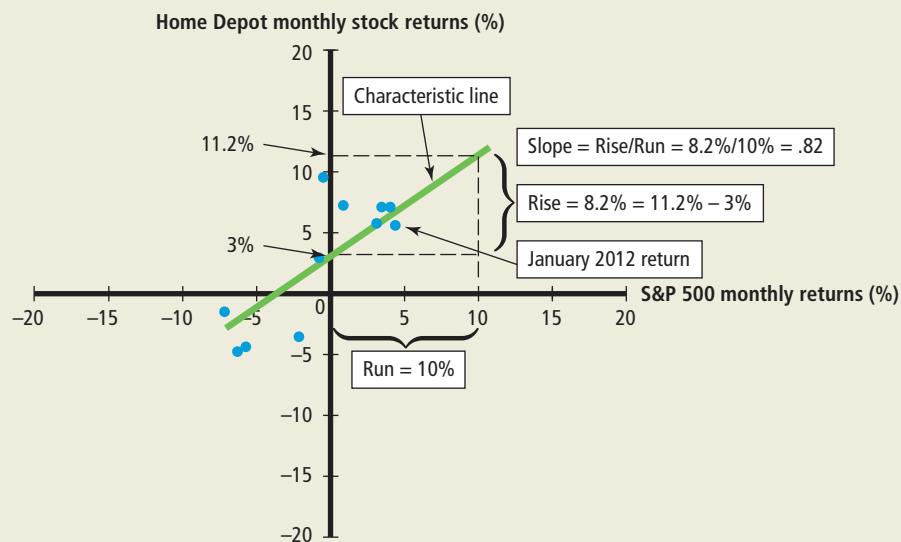
$$= \sqrt{\frac{(\text{return in month 1} - \text{average return})^2 + (\text{return in month 2} - \text{average return})^2 + \cdots + (\text{return in last month} - \text{average return})^2}{\text{number of monthly returns} - 1}} \quad (6-9)$$

In looking at Table 6-3 and Figure 6-4, we notice the following things about Home Depot's holding-period returns over the 12 months ending in June 2012.

1. Home Depot's shareholders realized significantly higher monthly holding-period returns on average than the general market, as represented by the Standard & Poor's 500 Index (S&P 500). Over the 12 months, Home Depot's stock had an average 3.33 percent monthly return compared only to 0.34 percent for the S&P 500 Index.
2. While Home Depot had higher average monthly holding-period returns than the general market (S&P 500 Index), at least for the 12-month period ending June 2012, the volatility (standard deviation) of the returns was only slightly higher for Home Depot than for the market—5.40 percent for Home Depot versus 5.23 percent for the S&P 500 Index.
3. We should also notice the tendency, although not perfect, of Home Depot's stock price to increase when the value of the S&P 500 Index increases and vice versa. This was the case in 10 of the 12 months. That is, there is a moderate positive relationship between Home Depot's stock returns and the S&P 500 Index returns.

With respect to our third observation, that there is a relationship between the stock returns of Home Depot and the S&P 500 Index, it is helpful to see this relationship by graphing Home Depot's returns against the S&P 500 Index returns. We provide such a graph in

FIGURE 6-5 Monthly Holding-Period Returns for Home Depot versus the S&P 500 Index, July 2011 Through June 2012



Calculating beta:

Visual—the slope of a straight line can be estimated visually by drawing a straight line that best “fits” the scatter of Home Depot’s stock returns and those of the market index. The beta coefficient then is the “rise over the run.” For example, when the S&P 500 Index is 10% shown on the horizontal axis (the run), Home Depot shown on the vertical axis (the rise) begins at 3% and goes to 11.2%, which is a rise of 8.2%. Thus, beta is the rise divided by the run, or $0.82 = 8.2\% \div 10\%$.

Financial calculator—financial calculators have built in functions for computing the beta coefficient. However, since the procedure varies from one calculator to another we do not present it here.

Excel—Excel’s Slope function can be used to calculate the slope, $=\text{slope}(\text{return values for Home Depot}, \text{return values for S\&P})$. For example, $=\text{slope}(c5:c16, e5:e16) = 0.82$.

Source: Data from Yahoo Finance

Figure 6-5. In the figure, we have plotted Home Depot’s returns on the vertical axis and the returns for the S&P 500 Index on the horizontal axis. Each of the 12 dots in the figure represents the returns of Home Depot and the S&P 500 Index for a particular month. For instance, the returns for January 2012 for Home Depot and the S&P 500 Index were 5.59 percent and 4.36 percent, respectively, which are noted in the figure.

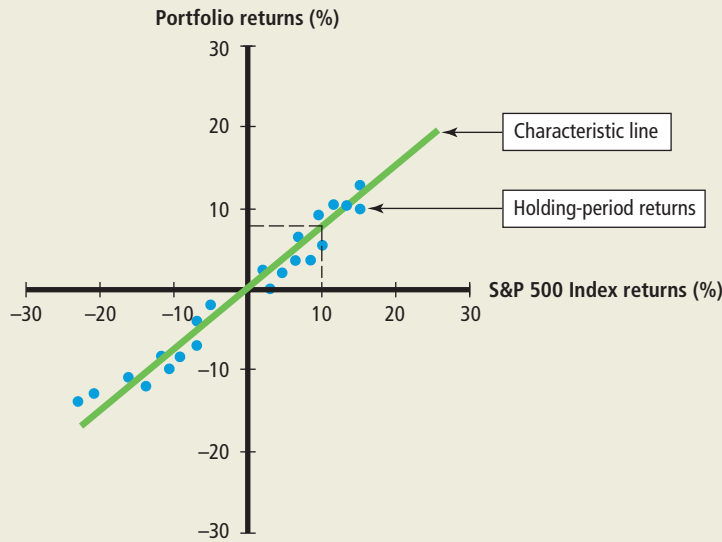
In addition to the dots in the graph, we have drawn a line of “best fit,” which we call the **characteristic line**. *The slope of the characteristic line measures the average relationship between a stock’s returns and those of the S&P 500 Index; or stated differently, the slope of the line indicates the average movement in a stock’s price in response to a movement in the S&P 500 Index price.* We can estimate the slope of the line visually by fitting a line that appears to cut through the middle of the dots. We then compare the rise (increase of the line on the vertical axis) to the run (increase on the horizontal axis). Alternatively, we can enter the return data into a financial calculator or in an Excel spreadsheet, which will calculate the slope based on statistical analysis. For Home Depot, the slope of the line is 0.82, which means that on average that as the market returns (S&P 500 Index returns) increase or decrease 1 percent, the return for Home Depot on average increases or decreases 0.82 percentage points.

We can also think of the 0.82 slope of the characteristic line as indicating that Home Depot’s returns are 0.82 times as volatile on average as those of the overall market (S&P 500 Index). This slope has come to be called **beta** (β) in investor jargon, and it *measures the average relationship between a stock’s returns and the market’s returns*. It is a term you will see almost any time you read an article written by a financial analyst about the riskiness of a stock.

Looking once again at Figure 6-5, we see that the dots (returns) are scattered all about the characteristic line—most of the returns do not fit neatly on the characteristic line. That is, the average relationship may be 0.82, but the variation in Home Depot’s returns is only partly explained by the stock’s average relationship with the S&P 500 Index. There are other driving forces unique to Home Depot that also affect the firm’s stock returns. (Earlier, we called this company-unique risk.) If we were, however, to diversify our

characteristic line the line of “best fit” through a series of returns for a firm’s stock relative to the market’s returns. The slope of the line, frequently called beta, represents the average movement of the firm’s stock returns in response to a movement in the market’s returns.

beta the relationship between an investment’s returns and the market’s returns. This is a measure of the investment’s nondiversifiable risk.

FIGURE 6-6 Holding-Period Returns for a Hypothetical Portfolio and the S&P 500 Index

Source: Data from Yahoo Finance

holdings and own, say, 20 stocks with betas of 0.82, we could essentially eliminate the variation about the characteristic line. That is, we would remove almost all the volatility in returns, except for that caused by the general market, which is represented by the slope of the line in Figure 6-5. If we plotted the returns of our 20-stock portfolio against the S&P 500 Index, the points in our new graph would fit nicely along a straight line with a slope of 0.82, which means that the beta (β) of the portfolio is also 0.82. The new graph would look something like the one shown in Figure 6-6. In other words, by diversifying our portfolio, we can essentially eliminate the variations about the characteristic line, leaving only the variation in returns for a company that comes from variations in the general market returns.

So beta (β)—the slope of the characteristic line—is a measure of a firm's market risk or systematic risk, which is the risk that remains for a company even after we have diversified

CAN YOU DO IT?

ESTIMATING BETA

Below, we provide the end-of-month prices for Toyota stock and the Standard & Poor's 500 Index for December 2011 through June 2012. Given the information, compute the following both for Toyota and the S&P 500: (1) the monthly holding-period returns, (2) the average monthly returns, and (3) the standard deviation of the returns. Next, graph the holding-period returns of Toyota on the vertical axis against the holding-period returns of the S&P 500 on the horizontal axis. Draw a line on your graph similar to what we did in Figure 6-5 to estimate the average relationship between the stock returns of Toyota and the returns of the overall market as represented by the S&P 500. What is the approximate slope of your line? What does this tell you?

(In working this problem, it would be easier if you used an Excel spreadsheet including the slope function.)

DATE	TOYOTA	S&P 500 INDEX
Dec-11	\$66.13	\$1,257.60
Jan-12	73.48	1,312.41
Feb-12	82.71	1,365.68
Mar-12	86.82	1,408.47
Apr-12	81.78	1,397.91
May-12	76.89	1,310.33
June-12	76.30	1,355.69

(The solution can be found on page 204.)

our portfolio. It is this risk—and only this risk—that matters for investors who have broadly diversified portfolios.

Although we have said that beta is a measure of a stock's systematic risk, how should we interpret a specific beta? For instance, when is a beta considered low and when is it considered high? In general, a stock with a beta of zero has no systematic risk; a stock with a beta of 1 has systematic or market risk equal to the “typical” stock in the marketplace; and a stock with a beta exceeding 1 has more market risk than the typical stock. Most stocks, however, have betas between 0.60 and 1.60.

We should also realize that calculating beta is not an exact science. The final estimate of a firm's beta is heavily dependent on one's methodology. For instance, it matters whether you use 24 months in your measurement or 60 months, as most professional investment companies do. Take our computation of Home Depot's beta. We said Home Depot's beta is 0.82, but Value Line, a well-known investment service, has estimated Home Depot's beta to be 0.95. Value Line's beta estimates for a number of firms are as follows:

COMPANY NAME	BETAS
Amazon	1.05
Apple	1.05
Coca-Cola	0.60
eBay	1.10
ExxonMobil	0.80
General Electric	1.20
IBM	0.85
Lowe's	0.95
Merck	0.80
Nike	0.85
PepsiCo	0.60
WalMart	0.60

EXAMPLE 6.2

Calculating monthly returns, average returns, standard deviation, and estimating beta

Given the following price data for Harley-Davidson and the S&P 500 Index, compare the returns and the volatility of the returns, and estimate the relationship of the returns for Harley-Davidson and the S&P 500 Index. What do you conclude?

DATE	HARLEY-DAVIDSON	S&P 500 INDEX
June-11	\$ 40.97	\$1,320.64
July-11	43.39	1,292.28
Aug-11	38.66	1,218.89
Sept-11	34.33	1,131.42
Oct-11	38.90	1,253.30
Nov-11	36.77	1,246.96
Dec-11	38.87	1,257.60
Jan-12	44.19	1,312.41
Feb-12	46.58	1,365.68
Mar-12	49.08	1,408.47
Apr-12	52.33	1,397.91
May-12	48.18	1,310.33
June-12	46.18	1,355.69

STEP 1: FORMULATE A DECISION STRATEGY

The following equations are needed to solve the problem:

Monthly holding-period returns:

$$\begin{aligned}\text{Monthly holding return} &= \frac{\text{price}_{\text{end of month}} - \text{price}_{\text{beginning of month}}}{\text{price}_{\text{beginning of month}}} \\ &= \frac{\text{price}_{\text{end of month}}}{\text{price}_{\text{beginning of month}}} - 1\end{aligned}$$

Average monthly returns:

$$\text{Average holding-period return} = \frac{\text{return in month 1} + \text{return in month 2} + \cdots + \text{return in last month}}{\text{number of monthly returns}}$$

Standard deviation of the returns:

Standard
deviation

$$= \sqrt{\frac{(\text{return in month 1} - \text{average return})^2 + (\text{return in month 2} - \text{average return})^2 + \cdots + (\text{return in last month} - \text{average return})^2}{\text{number of monthly returns} - 1}}$$

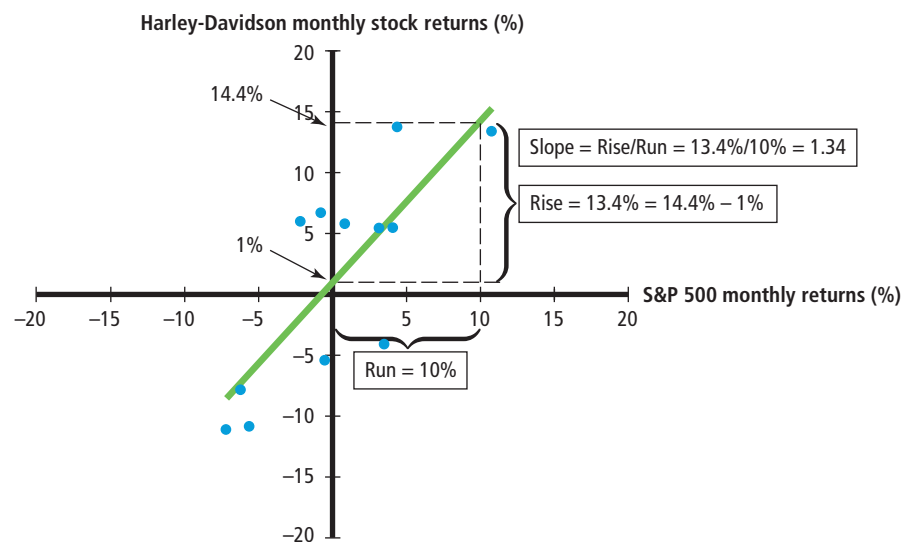
Determining the historical relationship between Harley-Davidson's stock returns and those of the S&P 500 Index requires our eye balling the line that best fits the average relationship, or using a financial calculator, or using a spreadsheet.

STEP 2: CRUNCH THE NUMBERS

Given the price data, we have calculated the monthly returns, the average returns, and the standard deviation of returns as follows, as well as the slope of the characteristic line (using a spreadsheet):

MONTH	HARLEY-DAVIDSON		S&P 500 INDEX	
	PRICE	RETURN	PRICE	RETURN
Jun-11	\$40.97		\$1,320.64	
Jul-11	43.39	5.9%	1,292.28	−2.1%
Aug-11	38.66	−10.9%	1,218.89	−5.7%
Sep-11	34.33	−11.2%	1,131.42	−7.2%
Oct-11	38.9	13.3%	1,253.30	10.8%
Nov-11	36.77	−5.5%	1,246.96	−0.5%
Dec-11	38.87	5.7%	1,257.60	0.9%
Jan-12	44.19	13.7%	1,312.41	4.4%
Feb-12	46.58	5.4%	1,365.68	4.1%
Mar-12	49.08	5.4%	1,408.47	3.1%
Apr-12	52.33	6.6%	1,397.91	−0.7%
May-12	48.18	−7.9%	1,310.33	−6.3%
Jun-12	46.18	−4.2%	1,355.69	3.5%
Average return		1.36%		0.34%
Standard deviation		8.87%		5.23%
Slope of the characteristic line				1.34

The relationship between the Harley-Davidson returns and the S&P 500 Index are shown in the graph below:



Source: Data from Yahoo Finance.

STEP 3: ANALYZE YOUR RESULTS

We can see that the average return for Harley-Davidson was significantly higher than the returns for the S&P 500. (Remember these are only monthly returns.) But the volatility of the returns for Harley-Davidson is also higher than for the S&P 500. Then when we regress the Harley-Davidson returns on the market (S&P 500) returns, we see that the average relationship is 1.34, which is a measure of systematic risk. That is, for every 1 percent that the market returns move up or down, Harley-Davidson's returns will on average move 1.34 percent.

To this point, we have talked about measuring an individual stock's beta. We will now consider how to measure the beta for a portfolio of stocks.

Measuring a Portfolio's Beta

What if we were to diversify our portfolio, as we have just suggested, but instead of acquiring stocks with the same beta as Home Depot (0.82), we buy 8 stocks with betas of 1.0 and 12 stocks with betas of 1.5. What would the beta of our portfolio become? As it works out, the **portfolio beta** is merely the average of the individual stock betas. It is a *weighted average of the individual securities' betas, with the weights being equal to the proportion of the portfolio invested in each security*. Thus, the beta (β) of a portfolio consisting of n stocks is equal to

$$\begin{aligned} \text{Portfolio beta} = & \left(\frac{\text{percentage of portfolio invested in asset 1}}{\text{in asset 1}} \times \frac{\text{beta for asset 1}}{(\beta_1)} \right) \\ & + \left(\frac{\text{percentage of portfolio invested in asset 2}}{\text{in asset 2}} \times \frac{\text{beta for asset 2}}{(\beta_2)} \right) \\ & + \dots + \left(\frac{\text{percentage of portfolio invested in asset } n}{\text{in asset } n} \times \frac{\text{beta for asset } n}{(\beta_n)} \right) \end{aligned} \quad (6-10)$$

So, assuming we bought equal amounts of each stock in our new 20-stock portfolio, the beta would simply be 1.3, calculated as follows:

$$\text{Portfolio beta} = \left(\frac{8}{20} \times 1.0 \right) + \left(\frac{12}{20} \times 1.50 \right) = 1.3$$

portfolio beta the relationship between a portfolio's returns and the market returns. It is a measure of the portfolio's nondiversifiable risk.

Thus, whenever the general market increases or decreases 1 percent, our new portfolio's returns would change 1.3 percent on average, which means that our new portfolio has more systematic, or market, risk than the market has as a whole.

We can conclude that the beta of a portfolio is determined by the betas of the individual stocks. If we have a portfolio consisting of stocks with low betas, then our portfolio will have a low beta. The reverse is true as well. Figure 6-7 presents these situations graphically.

Before leaving the subject of risk and diversification, we want to share a study that demonstrates the effects of diversifying our investments, not just across different stocks but also across different types of securities.

Risk and Diversification Demonstrated

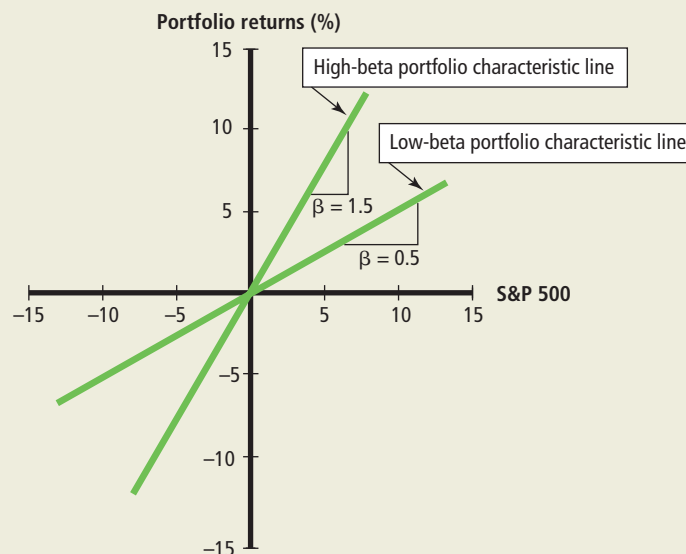
To this point, we have described the effect of diversification on risk and return in a general way. Also, when we spoke of diversification, we were diversifying by holding more stocks in our portfolio. Now let's look briefly on how risk and return change as we (1) diversify between two different types of assets—stocks and bonds, and (2) increase the length of time that we hold a portfolio of assets.

Notice that when we previously spoke about diversification, we were diversifying by holding more stock, but the portfolio still consisted of all stocks. Now we examine diversifying between a portfolio of stocks and a portfolio of bonds. Diversifying among different kinds of assets is called **asset allocation**, compared with diversification within the different asset classes, such as stocks, bonds, real estate, and commodities. We know from experience that the benefit we receive from diversifying is far greater through effective asset allocation than from astutely selecting individual stocks to include with an asset category.

Figure 6-8 presents the range of rolling returns for several mixtures of stocks and bonds depending on whether we held the investments 12 months, 60 months, or 120 months between 1926 and 2011. For the 12-month holding period, we would have purchased the investment at the beginning of each year and sold at the end of each year, repeating this process every year from 1926 to 2011. Then for the 60 months, we would have invested at the beginning of the year and held the investment for 60 months. In other words, we invested at the beginning of 1926 and sold at the end of 1930, then did the same for 1927–1931, repeating the process for all 60-month periods all the way through 2011. Finally, we would have invested at the beginning of each year, holding each investment for 120 months.

asset allocation identifying and selecting the asset classes appropriate for a specific investment portfolio and determining the proportions of those assets within the portfolio.

FIGURE 6-7 Holding-Period Returns: High- and Low-Beta Portfolios and the S&P 500 Index



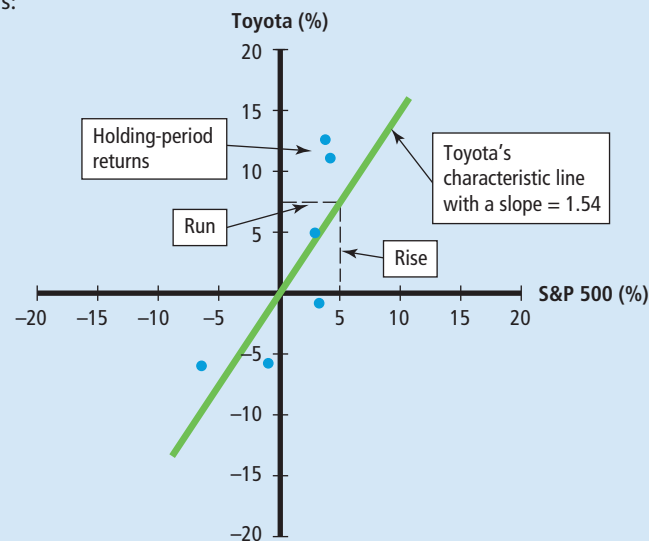
DID YOU GET IT?

ESTIMATING BETA

The holding-period returns, average monthly returns, and the standard deviations for Toyota and the S&P 500 data are as follows:

	TOYOTA		S&P 500	
	PRICES	RETURNS	PRICES	RETURNS
Dec-11	\$66.13		\$1,257.60	
Jan-12	73.48	11.11%	1,312.41	4.36%
Feb-12	82.71	12.56%	1,365.68	4.06%
Mar-12	86.82	4.97%	1,408.47	3.13%
Apr-12	81.78	−5.81%	1,397.91	−0.75%
May-12	76.89	−5.98%	1,310.33	−6.27%
Jun-12	76.30	−0.77%	1,355.69	3.46%
Average return		2.68%		1.33%
Standard deviation		8.16%		4.16%
Slope		1.54		

The graph would appear as follows:



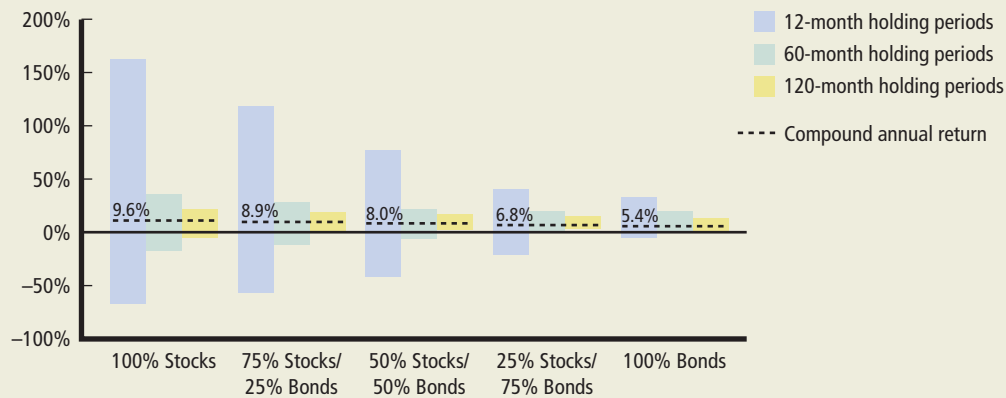
Source: Data from Yahoo Finance

The average relationship between Toyota's stock returns and the S&P 500's returns is estimated by the slope of the characteristic line in the graph above. Using a spreadsheet, we find the slope to be 1.54, where the rise of the line is about 7.5 percent relative to a run (horizontal axis) of 5 percent. Thus, Toyota's stock returns are more volatile than the market's returns. When the market rises (or falls) 1 percent, Toyota's stock will rise (or fall) 1.5 percent. (We should, however, be hesitant to draw any firm conclusions here, given the limited number of return observations.)



As we observe in Figure 6-8, moving from an all-stock portfolio to a mixture of stocks and bonds and finally to an all-bond portfolio reduces the variability of returns (our measure for risk) significantly along with declining average rates of return. Stated differently, if we want to increase our expected returns, we must assume more risk. That is, there is a clear relationship between risk and return, which reminds us of **Principle 3: Risk Requires a Reward**.

Equally important, how long we hold our investments matters greatly when it comes to reducing risk. As we move from 12 months to 60 months, and finally to 120 months, we see the range of return falling sharply, especially when we move from 12-month holding periods to 60-month holding periods. As a side note, notice that there has never been a time between 1926 and 2011 when investors lost money if they held an all-stock portfolio—the most risky portfolio—for 10 years. In other words, *the market rewards the patient investor*.

FIGURE 6-8 The Effect of Diversifying and Investing for Longer Periods of Time on Risk and Returns

Source: Data from Summary Statistics of Annual Total Returns: 1926 to 2011 Yearbook. Copyright © 2012 Ibbotson Associates, Inc.

In the next section, we complete our study of risk and returns by connecting risk—market or systematic risk—to the investor’s *required* rate of return. After all, although risk is an important issue, it is primarily important in its effect on the investor’s required rate of return.

We now can recap the financial decision tools used to measure market risk for a single investment and for a portfolio of investments. They can be shown as follows:

FINANCIAL DECISION TOOLS

Name of Tool	Formula	What It Tells You
Monthly holding-period return	$\text{Monthly holding-period return} = \frac{\text{Price}_{\text{end of month}} - \text{Price}_{\text{beginning of month}}}{\text{Price}_{\text{beginning of month}}}$ $= \frac{\text{Price}_{\text{end of month}}}{\text{Price}_{\text{beginning of month}}} - 1$	Calculates the percentage rate of return for a security held for a single month.
Average monthly holding-period rate of return	$\text{Average monthly holding-period return} = \frac{\text{return in month 1} + \text{return in month 2} + \cdots + \text{return in last month}}{\text{number of monthly returns}}$	Finds the average monthly holding-period return for a number of monthly returns.
Standard deviation of monthly holding-period returns	$\text{Standard Deviation} = \sqrt{\frac{(\text{return in month 1} - \text{average return})^2 + (\text{return in month 2} - \text{average return})^2 + \cdots + (\text{return in last month} - \text{average return})^2}{\text{number of monthly returns} - 1}}$	Measure of the volatility of monthly holding-period returns over a series of months.
Portfolio beta	$\text{Portfolio beta} = \left(\frac{\text{percentage of portfolio invested in asset 1}}{\text{percentage of portfolio invested in asset 1}} \times \frac{\text{beta for asset 1}}{(\beta_1)} \right) + \left(\frac{\text{percentage of portfolio invested in asset 2}}{\text{percentage of portfolio invested in asset 2}} \times \frac{\text{beta for asset 2}}{(\beta_2)} \right) + \cdots + \left(\frac{\text{percentage of portfolio invested in asset } n}{\text{percentage of portfolio invested in asset } n} \times \frac{\text{beta for asset } n}{(\beta_n)} \right)$	Finds the beta for an entire portfolio of stocks.

Concept Check

1. Give specific examples of systematic and unsystematic risk. How many different securities must be owned to essentially diversify away unsystematic risk?
2. What method is used to measure a firm's market risk?
3. After reviewing Figure 6-5, explain the difference between the plotted dots and the firm's characteristic line. What must be done to eliminate the variations?

b5 Explain the relationship between an investor's required rate of return on an investment and the riskiness of the investment.

required rate of return minimum rate of return necessary to attract an investor to purchase or hold a security.

risk-free rate of return the rate of return on risk-free investments. The interest rates on short-term U.S. government securities are commonly used to measure this rate.

risk premium the additional return expected for assuming risk.

capital asset pricing model (CAPM) an equation stating that the expected rate of return on an investment (in this case a stock) is a function of (1) the risk-free rate, (2) the investment's systematic risk, and (3) the expected risk premium for the market portfolio of all risky securities.

The Investor's Required Rate of Return

In this section, we examine the concept of the investor's required rate of return, especially as it relates to the riskiness of an asset, and then we see how the required rate of return might be measured.

The Required Rate of Return Concept

An investor's **required rate of return** can be defined as the *minimum rate of return necessary to attract an investor to purchase or hold a security*. This definition considers the investor's opportunity cost of funds of making an investment in *the next-best investment*. This forgone return is an opportunity cost of undertaking the investment and, consequently, is the investor's required rate of return. In other words, we invest with the intention of achieving a rate of return sufficient to warrant making the investment. The investment will be made only if the purchase price is low enough relative to expected future cash flows to provide a rate of return greater than or equal to our required rate of return.

To help us better understand the nature of an investor's required rate of return, we can separate the return into its basic components: the risk-free rate of return plus a risk premium. Expressed as an equation:

$$\text{Investor's required rate of return} = \text{risk-free rate of return} + \text{risk premium} \quad (6-11)$$

The **risk-free rate of return** is the *required rate of return, or discount rate, for riskless investments*. Typically, our measure for the risk-free rate of return is the U.S. Treasury bill rate. The **risk premium** is the *additional return we must expect to receive for assuming risk*. As the level of risk increases, we will demand additional expected returns. Even though we may or may not actually receive this incremental return, we must have reason to expect it; otherwise, why expose ourselves to the chance of losing all or part of our money?

To illustrate, assume you are considering the purchase of a stock that you believe will provide a 10 percent return over the next year. If the expected risk-free rate of return, such as the rate of return for 90-day Treasury bills, is 2 percent, then the risk premium you are demanding to assume the additional risk is 8 percent (10% – 2%).

Measuring the Required Rate of Return

We have seen that (1) systematic risk is the only relevant risk—the rest can be diversified away, and (2) the required rate of return, k , equals the risk-free rate plus a risk premium. We will now examine how we can estimate investors' required rates of return.

The finance profession has had difficulty in developing a practical approach to measure an investor's required rate of return; however, financial managers often use a method called the **capital asset pricing model (CAPM)**. The capital asset pricing model is *an equation that equates the expected rate of return on a stock to the risk-free rate plus a risk premium for the stock's systematic risk*. Although certainly not without its critics, the CAPM provides an intuitive approach for thinking about the return that an investor should require on an investment, given the asset's systematic or market risk.

Equation (6-11) above provides the natural starting point for measuring the investor's required rate of return and sets us up for using the CAPM. Rearranging this equation to solve for the risk premium we have

$$\text{Risk premium} = \text{investor's required rate of return, } r - \text{risk-free rate of return, } r_f \quad (6-12)$$

FINANCE AT WORK

DOES BETA ALWAYS WORK?

At the start of 1998, Apple Computer was in deep trouble. As a result, its stock price fluctuated wildly—far more than other computer firms, such as IBM. However, based on the capital asset pricing model (CAPM) and its measure of beta, the required return of Apple's investors would have been only 8 percent, compared with 12 percent for IBM's stockholders. Equally interesting, when Apple's situation improved in the spring of that year, and its share price became less volatile, Apple's investors, at least according to the CAPM, would have required a rate of return of 11 percent—a 3 percentage point increase from the earlier required rate of return. That is not what intuition would suggest should have happened.

So what should we think? Just when Apple's future was most in doubt and its shares most volatile, its beta was only 0.47, suggesting that Apple's stock was only half as volatile as the overall stock market. In reality, beta is meaningless here. The truth is that Apple was in such a dire condition that its stock price simply “decoupled” itself from the stock market. So, as IBM and its peer stock prices moved up and down with the rest of the market, Apple shares reacted solely to news about the company,

without regard for the market's movements. The stock's beta thus created the false impression that Apple shares were more stable than the stock market.

The lesson here is that beta may at times be misleading when used with individual companies. Instead, its use is far more reliable when applied to a portfolio of companies. If a firm that was interested in acquiring Apple Computer in 1998, for instance, had used the beta in computing the required rate of return for the acquisition, it would without a doubt have overvalued Apple.

So does that mean that CAPM is worthless? No, not as long as company-unique risk is not the main driving force in a company's stock price movements or if investors are able to diversify away specific company risk. Then they would bid up the price of such shares until they reflected only market risk. For example, a mutual fund that specializes in “distressed stocks” might purchase a number of “Apple Computer-type companies,” each with its own problems, but for different reasons. For such investors, betas work pretty well. Thus, the moral of the story is to exercise common sense and judgment when using a beta.

which simply says that the risk premium for a security equals the investor's required return less the risk-free rate existing in the market. For example, if the required return is 12 percent and the risk-free rate is 3 percent, the risk premium is 9 percent. Also, if the required return for the market portfolio is 10 percent, and the risk-free rate is 3 percent, the risk premium for the market would be 7 percent. This 7 percent risk premium would apply to any security having systematic (nondiversifiable) risk equivalent to the general market, or a beta (β) of 1.

In this same market, a security with a beta (β) of 2 should provide a risk premium of 14 percent, or twice the 7 percent risk premium existing for the market as a whole. Hence, in general, the appropriate required rate of return for a given security should be determined by

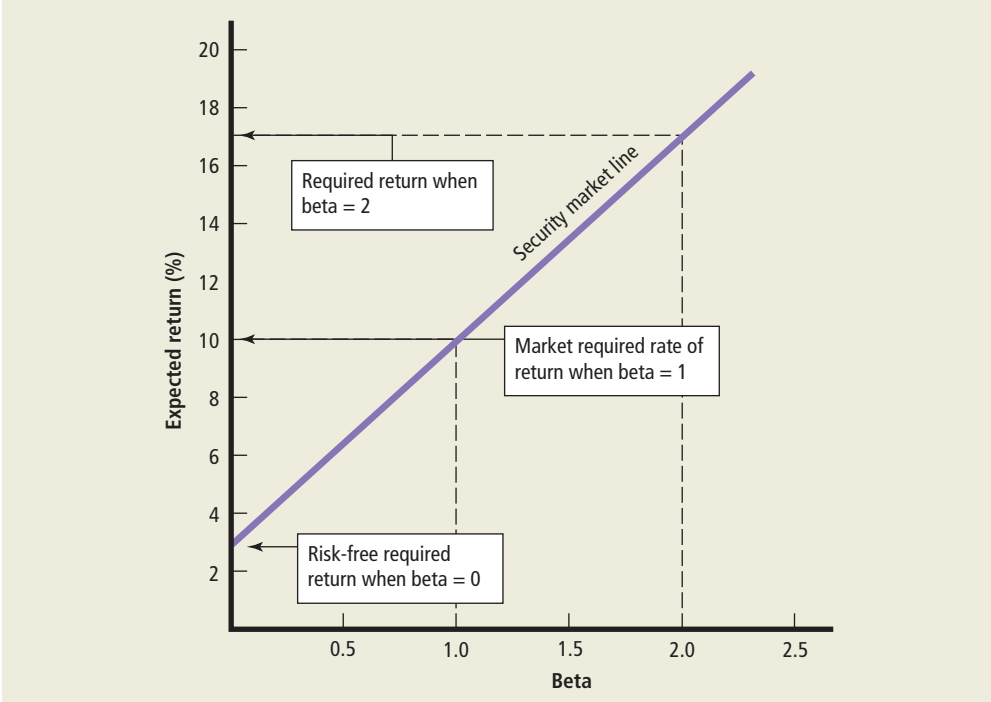
$$\begin{aligned} \text{Required return on} &= \text{risk free} \\ \text{security, } r & \quad \text{rate of return, } r_f \\ & + \text{beta for} \times \left(\text{required return} - \text{risk-free} \right) \\ & \quad \text{security, } \beta \quad \left(\text{on the market portfolio, } r_m \quad \text{rate of return, } r_f \right) \end{aligned} \quad (6-13)$$

Equation (6-13) is the CAPM. This equation designates the risk–return trade-off existing in the market, where risk is measured in terms of beta. Figure 6-9 graphs the CAPM as the **security market line**.⁹ The security market line is *the graphic representation of the CAPM. It is the line that shows the appropriate required rate of return given a stock's systematic risk*

security market line the return line that reflects the attitudes of investors regarding the minimum acceptable return for a given level of systematic risk associated with a security.

⁹Two key assumptions are made in using the security market line. First, we assume that the marketplace where securities are bought and sold is highly efficient. Market efficiency indicates that the price of an asset responds quickly to new information, thereby suggesting that the price of a security reflects all available information. As a result, the current price of a security is considered to represent the best estimate of its future price. Second, the model assumes that a perfect market exists. A perfect market is one in which information is readily available to all investors at a nominal cost. Also, securities are assumed to be infinitely divisible, with any transaction costs incurred in purchasing or selling a security being negligible. Furthermore, investors are assumed to be single-period wealth maximizers who agree on the meaning and significance of the available information. Finally, within the perfect market, all investors are *price takers*, which simply means that a single investor's actions cannot affect the price of a security. These assumptions are obviously not descriptive of reality. However, from the perspective of positive economics, the mark of a good theory is the accuracy of its predictions, not the validity of the simplifying assumptions that underlie its development.

FIGURE 6-9 Security Market Line



REMEMBER YOUR PRINCIPLES

The conclusion of the matter is that **Principle 3** is alive and well. It tells us, **Risk Requires a Reward**. That is, there is a risk–return trade-off in the market.

As presented in this figure, securities with betas equal to 0, 1, and 2, should have required rates of return as follows:

- If $\beta = 0$: required rate of return = $3\% + 0(10\% - 3\%) = 3\%$
- If $\beta = 1$: required rate of return = $3\% + 1(10\% - 3\%) = 10\%$
- If $\beta = 2$: required rate of return = $3\% + 2(10\% - 3\%) = 17\%$

where the risk-free rate is 3 percent, and the required return

for the market portfolio is 10 percent.

A large part of this chapter has been spent in explaining and measuring an investment’s expected rate of return and the corresponding amount of risk. However, one step remains—providing the financial tool or criterion to determine if the investment should be made. Only by knowing the investor’s required rate of return for a particular investment can we answer this question. The formulas used to answer this question were presented in this section and can be summarized as follows:

FINANCIAL DECISION TOOLS

Name of Tool	Formula	What It Tells You
Investor’s required rate of return	Investor’s required rate of return = risk-free rate of return + risk premium	Measures an investor’s required rate of return based on a risk-free rate plus a return premium for assuming risk
Risk premium	Risk premium = investor’s required rate of return, r – risk-free rate of return, r_f	Indicates the additional return an investor should require to assume additional risk
Investor’s required rate of return	$\text{Required return on security, } r = \text{risk free rate of return, } r_f + \text{beta for security, } \beta \times \left(\text{required return on the market portfolio, } r_m - \text{risk-free rate of return, } r_f \right)$	Calculates the rate of return an investor should require based on the capital asset pricing model, which only considers systematic risk to determine the appropriate risk premium

CAN YOU DO IT?

COMPUTING A REQUIRED RATE OF RETURN

Determine a fair expected (or required) rate of return for a stock that has a beta of 1.25 when the risk-free rate is 3 percent and the expected market return is 9 percent.

(The solution can be found below.)

Concept Check

1. How does opportunity cost affect an investor's required rate of return?
2. What are the two components of the investor's required rate of return?
3. How does beta fit into factoring the risk premium in the CAPM equation?
4. Assuming the market is efficient, what is the relationship between a stock's price and the security market line?

DID YOU GET IT?

COMPUTING A REQUIRED RATE OF RETURN

The appropriate rate of return would be:

$$\begin{aligned}\text{Required return} &= \text{risk-free rate} + [\text{beta} \times (\text{market return} - \text{risk-free rate})] \\ &= 3\% + [1.25 \times (9\% - 3\%)] \\ &= 10.5\%\end{aligned}$$

Chapter Summaries

In Chapter 2, we referred to the discount rate as the interest rate, or the opportunity cost of funds. At that point, we considered a number of important factors that influence interest rates, including the price of time, expected or anticipated inflation, the risk premium related to maturity (liquidity), and the variability of future returns. In this chapter, we have returned to our study of rates of return and looked ever so carefully at the relationship between risk and rates of return.

Define and measure the expected rate of return of an individual investment. (pgs 184–186)



SUMMARY: When we speak of “returns” on an investment, either we are talking about *historical* return, what we earned on an investment in the past, or *expected* returns, where we are attempting to determine the return we can “expect” to receive in the future. In a world of uncertainty, we cannot make forecasts with certitude. Thus, we must speak in terms of expected events. The expected return on an investment may therefore be stated as a weighted average of all possible returns, weighted by the probability that each will occur.

KEY TERMS

Holding-period return (historical or realized rate of return), page 184 The rate of return earned on an investment, which equals the dollar gain divided by the amount invested.

Expected rate of return, page 184 The arithmetic mean or average of all possible outcomes where those outcomes are weighted by the probability that each will occur.

KEY EQUATIONS

Holding-period dollar gain, DG

$$= \text{price}_{\text{end of period}} + \text{cash distribution (dividend)} - \text{price}_{\text{beginning of period}}$$

Holding-period rate of return, r

$$= \frac{\text{dollar gain}}{\text{price}_{\text{beginning of period}}} = \frac{\text{price}_{\text{end of period}} + \text{dividend} - \text{price}_{\text{beginning of period}}}{\text{price}_{\text{beginning of period}}}$$

$$\begin{aligned} \text{Expected cash flow, } \overline{CF} &= \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 1} & \text{of state 1} \\ (CF_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state 2} & \text{of state 2} \\ (CF_2) & (Pb_2) \end{array} \right) \\ &+ \dots + \left(\begin{array}{cc} \text{cash flow} & \text{probability} \\ \text{in state } n & \text{of state } n \\ (CF_n) & (Pb_n) \end{array} \right) \end{aligned}$$

$$\begin{aligned} \text{Expected rate of return, } \bar{r} &= \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 1} & \text{of state 1} \\ (r_1) & (Pb_1) \end{array} \right) + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state 2} & \text{of state 2} \\ (r_2) & (Pb_2) \end{array} \right) \\ &+ \dots + \left(\begin{array}{cc} \text{rate of return} & \text{probability} \\ \text{for state } n & \text{of state } n \\ (r_n) & (Pb_n) \end{array} \right) \end{aligned}$$

**2 Define and measure the riskiness of an individual investment.** (pgs 186–193)

SUMMARY: Risk associated with a single investment is the variability of cash flows or returns and can be measured by the standard deviation.

KEY TERMS

Risk, page 187 Potential variability in future cash flows.

Standard deviation, page 189 A statistical measure of the spread of a probability distribution calculated by squaring the

difference between each outcome and its expected value, weighting each value by its probability, summing over all possible outcomes, and taking the square root of this sum.

KEY EQUATION

$$\begin{aligned} \text{Variance in rates of return } (\sigma^2) &= \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 1} & \text{of return} \\ (r_1) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 1} \\ (Pb_1) \end{array} \right] \\ &+ \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state 2} & \text{of return} \\ (r_2) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state 2} \\ (Pb_2) \end{array} \right] \\ &+ \dots + \left[\left(\begin{array}{cc} \text{rate of return} & \text{expected rate} \\ \text{for state } n & \text{of return} \\ (r_n) & \bar{r} \end{array} \right)^2 \times \begin{array}{c} \text{probability} \\ \text{of state } n \\ (Pb_n) \end{array} \right] \end{aligned}$$

**3 Compare the historical relationship between risk and rates of return in the capital markets.** (pgs 193–194)

SUMMARY: Ibbotson Associates have provided us with annual rates of return earned on different types of security investments as far back as 1926. They summarize among other things, the annual returns for six portfolios of securities made up of

1. Common stocks of large companies
2. Common stocks of small firms
3. Long-term corporate bonds

4. Long-term U.S. government bonds
5. Intermediate-term U.S. government bonds
6. U.S. Treasury bills

A comparison of the annual rates of return for these respective portfolios for the years 1926 to 2011 shows a positive relationship between risk and return, with Treasury bills being least risky and common stocks of small firms being most risky. From the data, we are able to see the benefit of diversification in terms of improving the return–risk relationship. Also, the data clearly demonstrate that only common stock has in the long run served as an inflation hedge, and that the risk associated with common stock can be reduced if investors are patient in receiving their returns.

Explain how diversifying investments affects the riskiness and expected rate of return of a portfolio or combination of assets. (pgs 194–206)



SUMMARY: We made an important distinction between nondiversifiable risk and diversifiable risk. We concluded that the only relevant risk, given the opportunity to diversify our portfolio, is a security’s nondiversifiable risk, which we called by two other names: systematic risk and market-related risk.

KEY TERMS

Systematic risk (market risk or nondiversifiable risk), page 195 (1) The risk related to an investment return that cannot be eliminated through diversification. Systematic risk results from factors that affect all stocks. Also called market risk or nondiversifiable risk. (2) The risk of a project from the viewpoint of a well-diversified shareholder. This measure takes into account that some of the project’s risk will be diversified away as the project is combined with the firm’s other projects, and, in addition, some of the remaining risk will be diversified away by shareholders as they combine this stock with other stocks in their portfolios.

Unsystematic risk (company unique risk or diversifiable risk), page 195 The risk related to an investment return that can be eliminated through diversification. Unsystematic risk is the result of factors that are unique to the particular firm. Also called company-unique risk or diversifiable risk.

Characteristic line, page 198 The line of “best fit” through a series of returns for a firm’s stock relative to the market’s returns. The slope of the line, frequently called beta, represents the average movement of the firm’s stock returns in response to a movement in the market’s returns.

Beta, page 198 The relationship between an investment’s returns and the market’s returns. This is a measure of the investment’s nondiversifiable risk.

Portfolio beta, page 202 The relationship between a portfolio’s returns and the market returns. It is a measure of the portfolio’s nondiversifiable risk.

Asset allocation, page 203 Identifying and selecting the asset classes appropriate for a specific investment portfolio and determining the proportions of those assets within the portfolio.

KEY EQUATIONS

$$\begin{aligned}\text{Monthly holding return} &= \frac{\text{Price}_{\text{end of month}} - \text{Price}_{\text{beginning of month}}}{\text{Price}_{\text{beginning of month}}} \\ &= \frac{\text{Price}_{\text{end of month}}}{\text{Price}_{\text{beginning of month}}} - 1\end{aligned}$$

$$\text{Average holding-period return} = \frac{\text{return in month 1} + \text{return in month 2} + \cdots + \text{return in last month}}{\text{number of monthly returns}}$$

Standard deviation

$$= \sqrt{\frac{(\text{return in month 1} - \text{average return})^2 + (\text{return in month 2} - \text{average return})^2 + \cdots + (\text{return in last month} - \text{average return})^2}{\text{number of monthly returns} - 1}}$$

$$\begin{aligned} \text{Portfolio beta} = & \left(\begin{array}{cc} \text{percentage of} & \text{beta for} \\ \text{portfolio invested} & \text{asset 1} \\ \text{in asset 1} & (\beta_1) \end{array} \right) \\ & + \left(\begin{array}{cc} \text{percentage of} & \text{beta for} \\ \text{portfolio invested} & \text{asset 2} \\ \text{in asset 2} & (\beta_2) \end{array} \right) \\ & + \dots + \left(\begin{array}{cc} \text{percentage of} & \text{beta for} \\ \text{portfolio invested} & \text{asset } n \\ \text{in asset } n & (\beta_n) \end{array} \right) \end{aligned}$$



Explain the relationship between an investor's required rate of return on an investment and the riskiness of the investment. (pgs. 206–209)

SUMMARY: The capital asset pricing model provides an intuitive framework for understanding the risk–return relationship. The CAPM suggests that investors determine an appropriate required rate of return, depending upon the amount of systematic risk inherent in a security. This minimum acceptable rate of return is equal to the risk-free rate plus a risk premium for assuming risk.

KEY TERMS

Required rate of return, page 206 Minimum rate of return necessary to attract an investor to purchase or hold a security.

Risk-free rate of return, page 206 The rate of return on risk-free investments. The interest rates on short-term U.S. government securities are commonly used to measure this rate.

Risk premium, page 206 The additional return expected for assuming risk.

Capital asset pricing model (CAPM), page 206 An equation stating that the

expected rate of return on an investment (in this case a stock) is a function of (1) the risk-free rate, (2) the investment's systematic risk, and (3) the expected risk premium for the market portfolio of all risky securities.

Security market line, page 207 The return line that reflects the attitudes of investors regarding the minimum acceptable return for a given level of systematic risk associated with security.

KEY EQUATIONS

Investor's required rate of return = risk-free rate of return + risk premium

Risk premium = investor's required rate of return, r – risk-free rate of return, r_f

Required return on security, r = risk free rate of return, r_f + beta for security, β $\times$ $\left(\begin{array}{cc} \text{required return} & \text{risk-free} \\ \text{on the market portfolio, } r_m & \text{rate of return, } r_f \end{array} \right)$

Review Questions

All Review Questions are available in [MyFinanceLab](#).

- 6-1. a. What is meant by the investor's required rate of return?
b. How do we measure the riskiness of an asset?
c. How should the proposed measurement of risk be interpreted?
- 6-2. What is (a) unsystematic risk (company-unique or diversifiable risk) and (b) systematic risk (market or nondiversifiable risk)?
- 6-3. What is a beta? How is it used to calculate r , the investor's required rate of return?
- 6-4. What is the security market line? What does it represent?
- 6-5. How do we measure the beta of a portfolio?
- 6-6. If we were to graph the returns of a stock against the returns of the S&P 500 Index, and the points did not follow a very ordered pattern, what could we say about that stock? If the stock's returns tracked the S&P 500 returns very closely, then what could we say?

6-7. Over the past eight decades, we have had the opportunity to observe the rates of return and the variability of these returns for different types of securities. Summarize these observations.

6-8. What effect will diversifying your portfolio have on your returns?

Study Problems

All Study Problems are available in [MyFinanceLab](#).

6-1. (*Expected return and risk*) Universal Corporation is planning to invest in a security that has several possible rates of return. Given the following probability distribution of returns, what is the expected rate of return on the investment? Also compute the standard deviations of the returns. What do the resulting numbers represent?



PROBABILITY	RETURN
0.10	−10%
0.20	5%
0.30	10%
0.40	25%

6-2. (*Average expected return and risk*) Given the holding-period returns shown here, calculate the average returns and the standard deviations for the Kaifu Corporation and for the market.

MONTH	KAIFU CORP.	MARKET
1	4%	2%
2	6	3
3	0	1
4	2	−1

6-3. (*Expected rate of return and risk*) Carter Inc. is evaluating a security. Calculate the investment's expected return and its standard deviation.

PROBABILITY	RETURN
0.15	6%
0.30	9%
0.40	10%
0.15	15%

6-4. (*Expected rate of return and risk*) Summerville Inc. is considering an investment in one of two common stocks. Given the information that follows, which investment is better, based on the risk (as measured by the standard deviation) and return of each?

COMMON STOCK A		COMMON STOCK B	
PROBABILITY	RETURN	PROBABILITY	RETURN
0.30	11%	0.20	−5%
0.40	15%	0.30	6%
0.30	19%	0.30	14%
		0.20	22%

6-5. (*Standard deviation*) Given the following probabilities and returns for Mik's Corporation, find the standard deviation.

PROBABILITY	RETURNS
0.40	7%
0.25	4%
0.15	18%
0.20	10%

6-6. (*Expected return*) Go to <http://finance.yahoo.com>. Select the link Investing, and then choose Education. Go to Financial Glossary. Find the terms Expected Return and Expected Value in the glossary. What did you learn from these definitions?

6-7. Go to www.investopedia.com/university/beginner, where there is an article on “Investing 101: Introduction.” Read the article and explain what it says about risk tolerance.



6-8. Go to www.moneychimp.com. Select the link Volatility. Complete the retirement planning calculator, making the assumptions that you believe are appropriate for you. Then go to the Monte Carlo simulation calculator. Assume that you invest in large-company common stocks during your working years and then invest in long-term corporate bonds during retirement. Use the nominal average returns and standard deviations shown in Figure 6-2. What did you learn?



6-9. (*Holding-period returns*) From the price data that follow, compute the holding-period returns for periods 2 through 4.

PERIOD	STOCK PRICE
1	\$10
2	13
3	11
4	15

6-10. (*Computing holding-period returns*)

- a. From the price data here, compute the holding-period returns for Jazman and Solomon for periods 2 through 4.

PERIOD	JAZMAN	SOLOMON
1	\$ 9	\$27
2	11	28
3	10	32
4	13	29

- b. How would you interpret the meaning of a holding-period return?

6-11. (*Measuring risk and rates of return*)

- a. Given the holding-period returns shown here, compute the average returns and the standard deviations for the Zemin Corporation and for the market.

MONTH	ZEMIN CORP.	MARKET
1	6%	4%
2	3	2
3	−1	1
4	−3	−2
5	5	2
6	0	2

- b. If Zemin's beta is 1.54 and the risk-free rate is 4 percent, what would be an appropriate required return for an investor owning Zemin? (*Note:* Because the returns of Zemin Corporation are based on monthly data, you will need to annualize the returns to make them compatible with the risk-free rate. For simplicity, you can convert from monthly to yearly returns by multiplying the average monthly returns by 12.)
- c. How does Zemin's historical average return compare with the return you believe to be a fair return, given the firm's systematic risk?

6-12. (*Holding period dollar gain and return*) Suppose you purchased 16 shares of Disney stock for \$24.22 per share on May 1, 2012. On September 1 of the same year, you sold 12 shares of the stock for \$25.68. Calculate the holding-period dollar gain for the shares you sold, assuming no dividend was distributed, and the holding-period rate of return.

6-13. (*Asset allocation*) Go to <http://cgi.money.cnn.com/tools> and then go to the heading, investing and click the link Fix Your Asset Allocation. Answer the questions and click, Calculate. Try different options and see how the calculator suggests you allocate your investments.

6-14. (*Expected return, standard deviation, and capital asset pricing model*) The following are the end-of-month prices for both the Standard & Poor's 500 Index and Nike's common stock.



- a. Using the data here, calculate the holding-period returns for each of the months.

	NIKE	S&P 500 INDEX
2011		
June	\$89.98	\$1,320.64
July	90.15	1,292.28
August	86.65	1,218.89
September	85.51	1,131.42
October	96.35	1,253.30
November	96.18	1,246.96
December	96.37	1,257.60
2012		
January	103.99	1,312.41
February	107.92	1,365.68
March	108.44	1,408.47
April	111.87	1,397.91
May	108.18	1,310.33
June	98.45	1,355.69

- b. Calculate the average monthly return and the standard deviation for both the S&P 500 and Nike.
- c. Develop a graph that shows the relationship between the Nike stock returns and the S&P 500 Index. (Show the Nike returns on the vertical axis and the S&P 500 Index returns on the horizontal axis as done in Figure 6-5.)
- d. From your graph, describe the nature of the relationship between Nike stock returns and the returns for the S&P 500 Index.

6-15. (*Capital asset pricing model*) Using the CAPM, estimate the appropriate required rate of return for the three stocks listed here, given that the risk-free rate is 5 percent and the expected return for the market is 12 percent.

STOCK	BETA
A	0.75
B	0.90
C	1.40

6-16. (*Expected return, standard deviation, and the Capital Asset Pricing Model*) Below you have been provided the prices for Merck and the S&P 500 Index.

	MERCK	S&P 500 INDEX
2011		
June	\$32.27	\$1,320.64
July	32.75	1,292.28
August	30.87	1,218.89
September	29.49	1,131.42
October	31.83	1,253.30
November	29.59	1,246.96
December	30.33	1,257.60
2012		
January	31.60	1,312.41
February	35.74	1,365.68
March	36.90	1,408.47
April	41.02	1,397.91
May	39.19	1,310.33
June	42.50	1,355.69

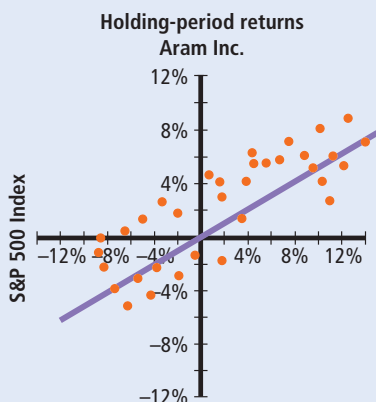
- Calculate the monthly holding-period returns for Merck and the S&P 500 Index.
- What are the average monthly returns and standard deviations for each?
- Graph the Merck returns against the S&P 500 Index returns.
- Use a spreadsheet to compute the slope of the characteristic line.
- Interpret your findings.

6-17. (*Security market line*)

- Determine the expected return and beta for the following portfolio:

STOCK	PERCENTAGE OF PORTFOLIO	BETA	EXPECTED RETURN
1	40%	1.00	12%
2	25	0.75	11
3	35	1.30	15

- Given the foregoing information, draw the security market line and show where the securities and portfolio fit on the graph. Assume that the risk-free rate is 2 percent and that the expected return on the market portfolio is 8 percent. How would you interpret these findings?



6-18. (*Required rate of return using CAPM*)

- Compute a fair rate of return for Intel common stock, which has a 1.2 beta. The risk-free rate is 2 percent, and the market portfolio (New York Stock Exchange stocks) has an expected return of 11 percent.
- Why is the rate you computed a fair rate?

6-19. (*Estimating beta*) From the graph in the margin relating the holding-period returns for Aram Inc. to the S&P 500 Index, estimate the firm's beta.

6-20. (*Capital asset pricing model*) Levine Manufacturing Inc. is considering several investments. The rate on Treasury bills is currently 2.75 percent, and the expected return for the market is 12 percent. What should be the required rates of return for each investment (using the CAPM)?

SECURITY	BETA
A	1.50
B	0.90
C	0.70
D	1.15

6-21. (*Capital asset pricing model*) MFI Inc. has a beta of 0.86. If the expected market return is 11.5 percent and the risk-free rate is 3 percent, what is the appropriate required return of MFI (using the CAPM)?

6-22. (*Capital asset pricing model*) The expected return for the general market is 12.8 percent, and the risk premium in the market is 9.3 percent. Tasaco, LBM, and Exxos have betas of 0.864, 0.693, and 0.575, respectively. What are the corresponding required rates of return for the three securities?

6-23. (*Portfolio beta and security market line*) You own a portfolio consisting of the stocks below:

STOCK	PERCENTAGE OF PORTFOLIO	BETA	EXPECTED RETURN
1	20%	1.00	12%
2	30	0.85	8
3	15	1.20	12
4	25	0.60	7
5	10	1.60	16

The risk-free rate is 3 percent. Also, the expected return on the market portfolio is 11 percent.

- Calculate the expected return of your portfolio. (*Hint:* The expected return of a portfolio equals the weighted average of the individual stocks' expected returns, where the weights are the percentage invested in each stock.)
- Calculate the portfolio beta.
- Given the foregoing information, plot the security market line on paper. Plot the stocks from your portfolio on your graph.
- From your plot in part (c), which stocks appear to be your winners and which ones appear to be losers?
- Why should you consider your conclusion in part (d) to be less than certain?

6-24. (*Portfolio beta*) Assume you have the following portfolio.

	STOCK WEIGHT	BETA
Apple	38%	1.50
Green Mountain Coffee	15%	1.44
Disney	27%	1.15
Target	20%	1.20

What is the portfolio's beta?

Mini Case

This Mini Case is available in [MyFinanceLab](#).

Note: Although not absolutely necessary, you are advised to use a computer spreadsheet to work the following problem.

- a. Use the price data from the table that follows for the Standard & Poor's 500 Index, Wal-mart, and Target to calculate the holding-period returns for the 24 months from July 2010 through June 2012.

MONTH	S&P 500	WAL-MART	TARGET
June-10	\$1,030.71	\$48.07	\$49.17
Jul-10	1,101.60	51.19	51.32
Aug-10	1,049.33	50.14	51.16
Sep-10	1,141.20	53.52	53.44
Oct-10	1,183.26	54.17	51.94
Nov-10	1,180.55	54.09	56.94
Dec-10	1,257.64	53.93	60.13
Jan-11	1,286.12	56.07	54.83
Feb-11	1,327.22	51.98	52.55
Mar-11	1,325.83	52.05	50.01
Apr-11	1,363.61	54.98	49.10
May-11	1,345.20	55.22	49.53
Jun-11	1,320.64	53.14	46.91
Jul-11	1,292.28	52.71	51.49
Aug-11	1,218.89	53.19	51.67
Sep-11	1,131.42	51.90	49.04
Oct-11	1,253.30	56.72	54.75
Nov-11	1,246.96	58.90	52.70
Dec-11	1,257.60	59.76	51.22
Jan-12	1,312.41	61.36	50.81
Feb-12	1,365.68	59.08	56.69
Mar-12	1,408.47	61.20	58.27
Apr-12	1,397.91	58.91	57.94
May-12	1,310.33	65.82	57.91
Jun-12	1,355.69	67.70	57.40

- b. Calculate the average monthly holding-period returns and the standard deviation of these returns for the S&P 500 Index, Wal-mart, and Target.
- c. Plot (1) the holding-period returns for Wal-mart against the Standard & Poor's 500 Index, and (2) the Target holding-period returns against the Standard & Poor's 500 Index. (Use Figure 6-5 as the format for your graph.)
- d. From your graphs in part (c), describe the nature of the relationship between the stock returns for Wal-mart and the returns for the S&P 500 Index. Make the same comparison for Target.
- e. Assume that you have decided to invest one-half of your money in Wal-mart and the remainder in Target. Calculate the monthly holding-period returns for your two-stock portfolio. (*Hint:* The monthly return for the portfolio is the average of the two stocks' monthly returns.)
- f. Plot the returns of your two-stock portfolio against the Standard & Poor's 500 Index as you did for the individual stocks in part (c). How does this graph compare to the graphs for the individual stocks? Explain the difference.
- g. The following table shows the returns on an *annualized* basis that were realized from holding long-term government bonds for the same period. Calculate the average *monthly* holding-period returns and the standard deviations of these returns. (*Hint:* You will need to convert the annual returns to monthly returns by dividing each return by 12 months.)

MONTH AND YEAR	ANNUALIZED RATE OF RETURN
Jun-10	4.03%
Jul-10	3.71%
Aug-10	3.82%
Sep-10	3.35%
Oct-10	3.40%
Nov-10	3.66%
Dec-10	3.95%
Jan-11	4.18%
Feb-11	4.37%
Mar-11	4.24%
Apr-11	4.27%
May-11	4.14%
Jun-11	3.83%
Jul-11	4.12%
Aug-11	3.72%
Sep-11	3.10%
Oct-11	2.51%
Nov-11	2.73%
Dec-11	2.82%
Jan-12	2.67%
Feb-12	2.65%
Mar-12	2.80%
Apr-12	3.00%
May-12	2.76%
Jun-12	2.13%

- h. Now assuming that you have decided to invest equal amounts of money in Wal-mart, Target, and long-term government securities, calculate the monthly returns for your three-asset portfolio. What is the average return and the standard deviation?
- i. Make a comparison of the average returns and the standard deviations for all the individual assets and the two portfolios that we designed. What conclusions can be reached by your comparison?
- j. According to Standard & Poor's, the betas for Wal-mart and Target are 0.45 and 0.60, respectively. Compare the meaning of these betas relative to the standard deviations calculated above.
- k. Assume that the current Treasury bill rate is 3 percent and that the expected market return is 10 percent. Given the betas for Wal-mart and Target in part (j), estimate an appropriate rate of return for the two firms.